

# Package: RelDists (via r-universe)

June 1, 2026

**Title** Estimation for some Reliability Distributions

**Version** 1.0.1

**Description** Parameters estimation and linear regression models for Reliability distributions families reviewed by Almalki & Nadarajah (2014) <doi:10.1016/j.res.2013.11.010> using Generalized Additive Models for Location, Scale and Shape, GAMLSS by Rigby & Stasinopoulos (2005) <doi:10.1111/j.1467-9876.2005.00510.x>.

**Depends** R (>= 3.5.0), survival, EstimationTools (>= 4.0.0)

**License** GPL-3

**URL** <https://fhernanb.github.io/RelDists/>

**BugReports** <https://github.com/fhernanb/RelDists/issues>

**Encoding** UTF-8

**RdMacros** Rdpack

**Imports** gamlss, gamlss.dist, Rdpack, zipfR, BBmisc, lamW, VGAM

**LazyData** true

**Suggests** knitr, rmarkdown, viridis, autoimage, gamlss.cens, V8, alr4

**VignetteBuilder** knitr

**Config/roxygen2/version** 8.0.0

**Config/pak/sysreqs** cmake make libicu-dev

**Repository** <https://fhernanb.r-universe.dev>

**Date/Publication** 2026-06-01 16:50:29 UTC

**RemoteUrl** <https://github.com/fhernanb/reldists>

**RemoteRef** HEAD

**RemoteSha** 54881a421e4619df1d084f27f837a21eed1bba6d

## Contents

AddW . . . . .	4
BGE . . . . .	6
BS . . . . .	7
BS10 . . . . .	10
BS11 . . . . .	12
BS12 . . . . .	13
BS13 . . . . .	15
BS2 . . . . .	17
BS3 . . . . .	19
BS4 . . . . .	21
BS5 . . . . .	24
BS6 . . . . .	25
BS7 . . . . .	27
CJ2 . . . . .	29
CS2e . . . . .	30
dAddW . . . . .	32
dBGE . . . . .	34
dBS . . . . .	36
dBS10 . . . . .	39
dBS11 . . . . .	41
dBS12 . . . . .	44
dBS13 . . . . .	46
dBS2 . . . . .	48
dBS3 . . . . .	51
dBS4 . . . . .	53
dBS5 . . . . .	55
dBS6 . . . . .	57
dBS7 . . . . .	60
dCJ2 . . . . .	62
dCS2e . . . . .	65
dEEG . . . . .	67
dEGG . . . . .	69
dEMWEx . . . . .	71
dEOFNH . . . . .	73
dEW . . . . .	75
dEXL . . . . .	76
dExW . . . . .	80
dExWALD . . . . .	81
dFWE . . . . .	84
dGammaW . . . . .	86
dGGD . . . . .	88
dGIW . . . . .	90
dGL2 . . . . .	92
dGMW . . . . .	94
dGWF . . . . .	96
dIW . . . . .	98

dKumIW	100
dLIN	102
dLW	104
dMOEIW	106
dMOEW	108
dMOK	109
dMW	111
dNEE	113
dOW	116
dPL	118
dQXGP	119
dRNMW	121
dRW	124
dSZMW	126
dWALD	127
dWG	130
dWGEE	132
dWP	133
EEG	135
EGG	137
EMWEx	139
EOFNH	140
equipment	142
estim_mu_sigma_GL2	143
EW	143
EXL	144
ExW	147
ExWALD	149
FWE	151
GammaW	152
GGD	154
GIW	155
GL2	157
GMW	160
initValuesOW	161
IW	163
KumIW	164
LIN	166
logLik_GL2	167
LW	168
mice	169
MOEIW	170
MOEW	171
MOK	173
MW	175
myOW_region	176
NEE	178
OW	181

param.startOW . . . . .	183
PL . . . . .	184
QXGP . . . . .	186
RW . . . . .	187
summary.initValOW . . . . .	189
SZMW . . . . .	189
WALD . . . . .	191
WG . . . . .	193
WGEE . . . . .	194
WP . . . . .	196

<b>Index</b>	<b>198</b>
--------------	------------

---

AddW	<i>The Additive Weibull family</i>
------	------------------------------------

---

## Description

The Additive Weibull distribution

## Usage

```
AddW(mu.link = "log", sigma.link = "log", nu.link = "log", tau.link = "log")
```

## Arguments

mu.link	defines the mu.link, with "log" link as the default for the mu parameter.
sigma.link	defines the sigma.link, with "log" link as the default for the sigma.
nu.link	defines the nu.link, with "log" link as the default for the nu parameter.
tau.link	defines the tau.link, with "log" link as the default for the tau parameter.

## Details

Additive Weibull distribution with parameters mu, sigma, nu and tau has density given by

$$f(x) = (\mu\nu x^{\nu-1} + \sigma\tau x^{\tau-1}) \exp(-\mu x^{\nu} - \sigma x^{\tau}),$$

for  $x > 0$ .

## Value

Returns a gamlss.family object which can be used to fit a AddW distribution in the gamlss() function.

## Author(s)

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

## References

- Almalki, S. J. (2018). A reduced new modified Weibull distribution. *Communications in Statistics-Theory and Methods*, 47(10), 2297-2313.
- Xie, M., & Lai, C. D. (1996). Reliability analysis using an additive Weibull model with bathtub-shaped failure rate function. *Reliability Engineering & System Safety*, 52(1), 87-93.

## See Also

[dAddW](#)

## Examples

```
# Example 1
# Generating some random values with
# known mu, sigma, nu and tau
# Will not be run this example because high number is cycles
# is needed in order to get good estimates
## Not run:
y <- rAddW(n=100, mu=1.5, sigma=0.2, nu=3, tau=0.8)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, tau.fo=~1, family='AddW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma, nu and tau
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))
exp(coef(mod, what='tau'))

## End(Not run)

# Example 2
# Generating random values under some model
# Will not be run this example because high number is cycles
# is needed in order to get good estimates
## Not run:
n <- 200
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(1.67 + -3 * x1)
sigma <- exp(0.69 - 2 * x2)
nu <- 3
tau <- 0.8
x <- rAddW(n=n, mu, sigma, nu, tau)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, tau.fo=~1, family=AddW,
              control=gamlss.control(n.cyc=5000, trace=FALSE))
```

```

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))
exp(coef(mod, what="tau"))

## End(Not run)

```

---

BGE

*The Beta Generalized Exponentiated family*


---

### Description

The Beta Generalized Exponentiated family

### Usage

```
BGE(mu.link = "log", sigma.link = "log", nu.link = "log", tau.link = "log")
```

### Arguments

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the mu parameter.
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the sigma.
<code>nu.link</code>	defines the <code>nu.link</code> , with "log" link as the default for the nu parameter.
<code>tau.link</code>	defines the <code>tau.link</code> , with "log" link as the default for the tau parameter.

### Details

The Beta Generalized Exponentiated distribution with parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$  has density given by

$$f(x) = \frac{\nu\tau}{B(\mu, \sigma)} \exp(-\nu x) (1 - \exp(-\nu x))^{\tau\mu-1} (1 - (1 - \exp(-\nu x))^{\tau})^{\sigma-1},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$ ,  $\nu > 0$  and  $\tau > 0$ .

### Value

Returns a `gamlss.family` object which can be used to fit a BGE distribution in the `gamlss()` function.

### Author(s)

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

### References

- Almalki, S. J. (2018). A reduced new modified Weibull distribution. *Communications in Statistics-Theory and Methods*, 47(10), 2297-2313.
- Barreto-Souza, W., Santos, A. H., & Cordeiro, G. M. (2010). The beta generalized exponential distribution. *Journal of statistical Computation and Simulation*, 80(2), 159-172.

**See Also**[dBGE](#)**Examples**

```

# Generating some random values with
# known mu, sigma, nu and tau
y <- rBGE(n=100, mu = 1.5, sigma =1.7, nu=1, tau=1)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, tau.fo=~1, family=BGE,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma, nu and tau
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))
exp(coef(mod, what='tau'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(0.5 - x1)
sigma <- exp(0.8 - x2)
nu <- 1
tau <- 1
x <- rBGE(n=n, mu, sigma, nu, tau)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, tau.fo=~1, family=BGE,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))
exp(coef(mod, what="tau"))

```

**Description**

The function `BS()` defines The Birnbaum-Saunders, a two parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

**Usage**

```
BS(mu.link = "log", sigma.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.

**Details**

The Birnbaum-Saunders with parameters `mu` and `sigma` has density given by

$$f(x|\mu, \sigma) = \frac{x^{-3/2}(x+\mu)}{2\sigma\sqrt{2\pi\mu}} \exp\left(\frac{-1}{2\sigma^2}\left(\frac{x}{\mu} + \frac{\mu}{x} - 2\right)\right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization  $\mu$  is the median of  $X$ ,  $E(X) = \mu(1 + \sigma^2/2)$  and  $Var(X) = (\mu\sigma)^2(1 + 5\sigma^2/4)$ . The functions proposed here corresponds to the functions created by Roquim et al. (2021) with minor modifications to obtain correct log-likelihoods and random samples.

**Value**

Returns a `gamlss.family` object which can be used to fit a BS distribution in the `gamlss()` function.

**References**

Birnbaum, Z.W. and Saunders, S.C. (1969a). A new family of life distributions. *J. Appl. Prob.*, 6, 319–327.

Roquim, F. V., Ramires, T. G., Nakamura, L. R., Righetto, A. J., Lima, R. R., & Gomes, R. A. (2021). Building flexible regression models: including the Birnbaum-Saunders distribution in the `gamlss` package. *Semina: Ciências Exatas e Tecnológicas*, 42(2), 163-168.

**See Also**

[dBS](#)

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
y <- rBS(n=100, mu=0.75, sigma=1.3)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))
```

```
# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y as BS
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(1.45 - 3 * x1)
  sigma <- exp(2 - 1.5 * x2)
  y <- rBS(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(123)
dat <- gendat(n=300)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
              family=BS, data=dat)

summary(mod2)

# Example 3
# Fatigue life (T) measures in cycles ( $\times 10^{-3}$ ) of n 101
# aluminum coupons (specimens) of type 6061-T6.
# Taken from Leiva et al. (2006) page 37.
# https://journal.r-project.org/articles/RN-2006-033/RN-2006-033.pdf

y <- c(70, 90, 96, 97, 99, 100, 103, 104,
      104, 105, 107, 108, 108, 108, 109, 109,
      112, 112, 113, 114, 114, 114, 116, 119,
      120, 120, 120, 121, 121, 123, 124, 124,
      124, 124, 124, 128, 128, 129, 129, 130,
      130, 130, 131, 131, 131, 131, 131, 132,
      132, 132, 133, 134, 134, 134, 134, 134,
      136, 136, 137, 138, 138, 138, 139, 139,
      141, 141, 142, 142, 142, 142, 142, 142,
      144, 144, 145, 146, 148, 148, 149, 151,
      151, 152, 155, 156, 157, 157, 157, 157,
      158, 159, 162, 163, 163, 164, 166, 166,
      168, 170, 174, 196, 212)

mod3 <- gamlss(y~1, sigma.fo=~1, family=BS)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod3, what="mu"))
exp(coef(mod3, what="sigma"))

# Example 4
# Aggregate payments by the insurer
# in thousand Skr (Swedish currency).
# Taken from Balakrishnan and Kundu (2019) page 65.
# https://onlinelibrary.wiley.com/doi/abs/10.1002/asmb.2348
```

```

y <- c(5014, 5855, 6486, 6540, 6656, 6656, 7212, 7541, 7558,
      7797, 8546, 9345, 11762, 12478, 13624, 14451,
      14940, 14963, 15092, 16203, 16229, 16730, 18027,
      18343, 19365, 21782, 24248, 29069, 34267, 38993)

y <- y/10000

mod4 <- gamlss(y~1, sigma.fo=~1, family=BS)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod4, what="mu"))
exp(coef(mod4, what="sigma"))

```

BS10

*The Birnbaum-Saunders family - Santos-Neto et al. (2012) (P8 Based on the first Tweedie)*

### Description

The function `BS10()` defines the Birnbaum-Saunders distribution, a two-parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

### Usage

```
BS10(mu.link = "log", sigma.link = "log")
```

### Arguments

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter (representing  $\tau$ ).

`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma` parameter (representing  $\omega$ ).

### Details

The Birnbaum-Saunders distribution with parameters `mu` and `sigma` (where `mu` represents  $\tau$  and `sigma` represents  $\omega$ ) has density given by

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\sigma}{\mu^2} \left[\frac{2x}{\mu^2} + \frac{\mu^2}{2x} - 2\right]\right) \frac{[2x + \mu^2]\sqrt{\sigma}}{2\mu^2\sqrt{x^3}}$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \frac{\mu^2}{2} + \frac{\mu^4}{8\sigma}$  and  $Var(X) = \frac{\mu^6}{8\sigma} + \frac{5\mu^8}{64\sigma^2}$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a BS10 distribution in the `gamlss()` function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

**See Also**

[dBS10](#).

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(12345)
y <- rBS10(n=100, mu=0.75, sigma=5)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS10)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y ~ BS10
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(0.5 - 1.6 * x1)      # Aprox 0.75
  sigma <- exp(2.2 - 1.2 * x2)  # Aprox 5
  y <- rBS10(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(123)
dat <- gendat(n=200)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
              family=BS10, data=dat)
```

summary(mod2)

---

BS11	<i>The Birnbaum-Saunders family - Santos-Neto et al. (2012) (P9 Based on the second Tweedie)</i>
------	--

---

### Description

The function `BS11()` defines the Birnbaum-Saunders distribution, a two-parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

### Usage

```
BS11(mu.link = "log", sigma.link = "log")
```

### Arguments

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the <code>mu</code> parameter (representing $\beta$ ).
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the <code>sigma</code> parameter (representing $\omega$ ).

### Details

The Birnbaum-Saunders distribution with parameters `mu` and `sigma` (where `mu` represents  $\beta$  and `sigma` represents  $\omega$ ) has density given by

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\sigma}{2\mu} \left[\frac{x}{\mu} + \frac{\mu}{x} - 2\right]\right) \frac{[x+\mu]\sqrt{\sigma}}{2\mu\sqrt{x^3}}$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \mu + \frac{\mu^2}{2\sigma}$  and  $Var(X) = \frac{\mu^3}{\sigma} + \frac{5\mu^4}{4\sigma^2}$ .

### Value

Returns a `gamlss.family` object which can be used to fit a BS11 distribution in the `gamlss()` function.

### Author(s)

David Villegas Ceballos, <david.villegas1@udea.edu.co>

### References

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

**See Also**

[dBS11](#).

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(1234)
y <- rBS11(n=100, mu=1, sigma=12)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS11)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y ~ BS11
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(0.5 - 1 * x1)      # Aprox 1
  sigma <- exp(1.9 + 1.2 * x2) # Aprox 12
  y <- rBS11(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(123)
dat <- gendat(n=200)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
               family=BS11, data=dat)

summary(mod2)
```

---

BS12

*The Birnbaum-Saunders family - Santos-Neto et al. (2012) (P10 Based on the third Tweedie)*

---

**Description**

The function `BS12()` defines the Birnbaum-Saunders distribution, a two-parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

**Usage**

```
BS12(mu.link = "log", sigma.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter (representing the scale  $\beta$ ).

`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma` parameter (representing the shape  $\psi$ ).

**Details**

The Birnbaum-Saunders distribution with parameters `mu` and `sigma` (where `mu` represents  $\beta$  and `sigma` represents  $\psi$ ) has density given by

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\sigma}{2} \left[\frac{x}{\mu} + \frac{\mu}{x} - 2\right]\right) \frac{[x+\mu]\sqrt{\sigma}}{2\sqrt{\mu x^3}}$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \mu + \frac{\mu}{2\sigma}$  and  $Var(X) = \frac{\mu^2}{\sigma} + \frac{5\mu^2}{4\sigma^2}$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a BS12 distribution in the `gamlss()` function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

**See Also**

[dBS12](#).

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(12345)
y <- rBS12(n=100, mu=1, sigma=30)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS12)

# Extracting the fitted values for mu and sigma
# using the inverse link function
```

```

exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with  $Y \sim \text{BS12}$ 
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(0.5 - 1 * x1)      # Aprox 1
  sigma <- exp(2.2 + 2.4 * x2) # Aprox 30
  y <- rBS12(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(123)
dat <- gendat(n=200)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
               family=BS12, data=dat)

summary(mod2)

```

---

BS13

*The Birnbaum-Saunders family - Santos-Neto et al. (2012) (P11 Based on the fourth Tweedie)*

---

## Description

The function `BS13()` defines the Birnbaum-Saunders distribution, a two-parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

## Usage

```
BS13(mu.link = "log", sigma.link = "log")
```

## Arguments

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the mu parameter (representing the scale $\omega$ ).
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the sigma parameter (representing the shape $\psi$ ).

### Details

The Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$  (where  $\mu$  represents  $\omega$  and  $\sigma$  represents  $\psi$ ) has density given by

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\sigma}{2} \left[\frac{x\sigma}{\mu} + \frac{\mu}{x\sigma} - 2\right]\right) \frac{[x\sigma + \mu]}{2\sqrt{\mu x^3}}$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \frac{\mu}{\sigma} + \frac{\mu}{2\sigma^2}$  and  $Var(X) = \frac{\mu^2}{\sigma^3} + \frac{5\mu^2}{4\sigma^4}$ .

### Value

Returns a `gamlss.family` object which can be used to fit a BS13 distribution in the `gamlss()` function.

### Author(s)

David Villegas Ceballos, <david.villegas1@udea.edu.co>

### References

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

### See Also

[dBS13](#).

### Examples

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(123456)
y <- rBS13(n=500, mu=5, sigma=30)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS13,
               control=gamlss.control(n.cyc=300, trace=TRUE))

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y ~ BS13
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
```

```

mu <- exp(1.1 + 1.1 * x1)      # Aprox 5
sigma <- exp(2.2 + 2.4 * x2)  # Aprox 30
y <- rBS13(n=n, mu=mu, sigma=sigma)
data.frame(y=y, x1=x1, x2=x2)
}

set.seed(123)
dat <- gendat(n=200)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
              family=BS13, data=dat,
              control=gamlss.control(n.cyc=500, trace=TRUE))

summary(mod2)

```

## Description

The function `BS2()` defines The Birnbaum-Saunders, a two parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

## Usage

```
BS2(mu.link = "log", sigma.link = "log")
```

## Arguments

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.

## Details

The Birnbaum-Saunders with parameters `mu` and `sigma` has density given by

$$f(x|\mu, \sigma) = \frac{\exp(\frac{\sigma}{2})\sqrt{\sigma+1}}{4\sqrt{\pi}\mu x^{3/2}} \left[ x + \frac{\mu\sigma}{\sigma+1} \right] \exp\left(\frac{-\sigma}{4} \left( \frac{x(\sigma+1)}{\mu\sigma} + \frac{\mu\sigma}{x(\sigma+1)} \right)\right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization  $E(X) = \mu$  and  $Var(X) = \frac{\mu^2(2\sigma+5)}{(\sigma+1)^2}$ .

## Value

Returns a `gamlss.family` object which can be used to fit a BS2 distribution in the `gamlss()` function.

## References

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Barros, M. (2014). A reparameterized Birnbaum-Saunders distribution and its moments, estimation and applications. *REVSTAT-Statistical Journal*, 12(3), 247-272.

**See Also**

[dBS2](#).

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
y <- rBS2(n=50, mu=5, sigma=3)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS2)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y ~ BS2
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(1.45 - 3 * x1)
  sigma <- exp(2 - 1.5 * x2)
  y <- rBS2(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(123)
dat <- gendat(n=100)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
               family=BS2, data=dat)

summary(mod2)

# Example 3
# Household expenditures for food in the United States (US) expressed
# in thousands of US dollars (M$)
# Santos-Neto et al. (2014) page 266.

y <- c(15.998, 16.652, 21.741, 7.431, 10.481, 13.548, 23.256, 17.976,
       14.161, 8.825, 14.184, 19.604, 13.728, 21.141, 17.446, 9.629,
       14.005, 9.160, 18.831, 7.641, 13.882, 9.670, 21.604, 10.866,
       28.980, 10.882, 18.561, 11.629, 18.067, 14.539, 19.192, 25.918,
       28.833, 15.869, 14.910, 9.550, 23.066, 14.751)

mod3 <- gamlss(y~1, sigma.fo=~1, family=BS2)
```

```

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod3, what="mu"))
exp(coef(mod3, what="sigma"))

# Example 4
# lifetimes of 6061-T6 aluminum coupons expressed in cycles (×10-3)
# at a maximum stress level of 3.1 psi (×104), until the failure to occur.
# Santos-Neto et al. (2014) page 267.

y <- c(70, 90, 96, 97, 99, 100, 103, 104, 104, 105, 107, 108, 108, 108, 109,
      109, 112, 112, 113, 114, 114, 114, 116, 119, 120, 120, 120, 121, 121,
      123, 124, 124, 124, 124, 124, 128, 128, 129, 129, 130, 130, 130, 131,
      131, 131, 131, 131, 132, 132, 132, 133, 134, 134, 134, 134, 134, 136,
      136, 137, 138, 138, 138, 139, 139, 141, 141, 142, 142, 142, 142, 142,
      142, 144, 144, 145, 146, 148, 148, 149, 151, 151, 152, 155, 156, 157,
      157, 157, 157, 158, 159, 162, 163, 163, 164, 166, 166, 168, 170, 174,
      196, 212)

mod4 <- gamlss(y~1, sigma.fo=~1, family=BS2)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod4, what="mu"))
exp(coef(mod4, what="sigma"))

```

## Description

The function `BS3()` defines The Birnbaum-Saunders, a two parameter distribution, for a `gamlss` family object to be used in GAMLSS fitting using the function `gamlss()`.

## Usage

```
BS3(mu.link = "log", sigma.link = "logit")
```

## Arguments

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "logit" link as the default for the `sigma`.

## Details

The Birnbaum-Saunders with parameters `mu` and `sigma` has density given by

$$f(x|\mu, \sigma) = \frac{(1-\sigma)y+\mu}{2\sqrt{2\pi\mu\sigma(1-\sigma)y^{3/2}}} \exp\left[\frac{-1}{2\sigma}\left(\frac{(1-\sigma)y}{\mu} + \frac{\mu}{(1-\sigma)y} - 2\right)\right]$$

for  $x > 0$ ,  $\mu > 0$  and  $0 < \sigma < 1$ . In this parameterization  $Mode(X) = \mu$  and  $Var(X) = (\mu\sigma)^2(1 + 5\sigma^2/4)$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a BS3 distribution in the `gamlss()` function.

**References**

Bourguignon, M., & Gallardo, D. I. (2022). A new look at the Birnbaum–Saunders regression model. *Applied Stochastic Models in Business and Industry*, 38(6), 935-951.

**See Also**

[dBS3](#).

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
y <- rBS3(n=50, mu=2, sigma=0.2)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS3)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y ~ BS3
## Not run:
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(1.45 - 3 * x1)
  inv_logit <- function(x) 1 / (1 + exp(-x))
  sigma <- inv_logit(2 - 1.5 * x2)
  y <- rBS3(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(1234)
dat <- gendat(n=100)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
              family=BS3, data=dat,
              control=gamlss.control(n.cyc=100))

summary(mod2)
```

```
## End(Not run)

# Example 3
# The response variable is the ratio between the average
# rent per acre planted with alfalfa and the corresponding
# average rent for other agricultural uses. The density of
# dairy cows (X2, number per square mile) is the explanatory variable.
library(alr4)
data("landrent")

landrent$ratio <- landrent$Y / landrent$X1

with(landrent, plot(x=X2, y=ratio))

mod3 <- gamlss(ratio~X2, sigma.fo=~X2,
               data=landrent, family=BS3)

summary(mod3)
logLik(mod3)
```

---

BS4

*The Birnbaum-Saunders family - Ahmed et al. (2008)*


---

## Description

The function `BS4()` defines the Birnbaum-Saunders distribution, a two-parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

## Usage

```
BS4(mu.link = "log", sigma.link = "log")
```

## Arguments

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma` parameter.

## Details

The Birnbaum-Saunders distribution with parameters `mu` and `sigma` has density given by

$$f(x|\mu, \sigma) = \frac{1}{2\sqrt{2\pi}} \left[ \frac{\sigma}{x\sqrt{x}} + \frac{\mu}{\sqrt{x}} \right] \exp \left( -\frac{1}{2} \left[ \frac{\sigma}{\sqrt{x}} - \mu\sqrt{x} \right]^2 \right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \frac{\sigma\mu+1/2}{\mu^2}$  and  $Var(X) = \frac{\sigma\mu+5/4}{\mu^4}$ .

## Value

Returns a `gamlss.family` object which can be used to fit a BS4 distribution in the `gamlss()` function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Ahmed, S. E., Budsaba, K., Lisawadi, S., & Volodin, A. (2008). Parametric estimation for the Birnbaum-Saunders lifetime distribution based on a new parametrization. *Thailand Statistician*, 6(2), 213-240.

**See Also**

[dBS4](#).

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(123)
y <- rBS4(n=50, mu=2, sigma=0.2)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS4)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y ~ BS4
## Not run:
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(1.45 - 3 * x1)
  sigma <- exp(2 - 1.5 * x2)
  y <- rBS4(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(1234)
dat <- gendat(n=100)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
              family=BS4, data=dat,
              control=gamlss.control(n.cyc=100))

summary(mod2)
```

```

## End(Not run)

# Example 3
# Taken from Ahmed et al. (2008) on page 227
# The response variable is the fatigue
# life of 6061-T6 aluminum coupons.
## Not run:
y <- c(
  70, 90, 96, 97, 99, 100, 103, 104,
  104, 105, 107, 108, 108, 108, 109, 109,
  112, 112, 113, 114, 114, 114, 116, 119,
  120, 120, 120, 121, 121, 123, 124, 124,
  124, 124, 124, 128, 128, 129, 130, 130,
  130, 130, 131, 131, 131, 131, 131, 132,
  132, 132, 133, 134, 134, 134, 134, 134,
  136, 136, 137, 138, 138, 138, 139, 139,
  141, 141, 142, 142, 142, 142, 142, 142,
  144, 144, 145, 146, 148, 148, 149, 151,
  151, 152, 155, 156, 157, 157, 157, 157,
  158, 159, 162, 163, 163, 164, 166, 166,
  168, 170, 174, 196, 212
)

mod3 <- gamlss(y~1, family=BS4,
               control=gamlss.control(n.cyc=3000))

summary(mod3)
exp(coef(mod3, what="mu"))
exp(coef(mod3, what="sigma"))

plot(density(y))
curve(dBS4(x, mu=0.5145121, sigma=67.81761),
      add=TRUE, col="tomato", lwd=2)
legend("topright", legend=c("Empirical", "Estimated"),
      col=c("black", "tomato"), lty=1)

## End(Not run)

# Example 4
# Taken from Ahmed et al. (2008) on page 228
# The response variable is the fatigue life in
# hours of 10 bearings of a certain type.
## Not run:
y <- c(152.7, 172.0, 172.5, 173.5, 193.0,
      204.7, 216.5, 234.9, 262.6, 422.6)

mod4 <- gamlss(y~1, family=BS4,
               control=gamlss.control(n.cyc=3000))

summary(mod4)
exp(coef(mod4, what="mu"))
exp(coef(mod4, what="sigma"))

```

```
## End(Not run)
```

---

BS5	<i>The Birnbaum-Saunders family - Santos-Neto et al. (2012) (P3 Based on GLM)</i>
-----	---

---

### Description

The function `BS5()` defines the Birnbaum-Saunders distribution, a two-parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

### Usage

```
BS5(mu.link = "log", sigma.link = "log")
```

### Arguments

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma` parameter.

### Details

The Birnbaum-Saunders distribution with parameters `mu` and `sigma` (where `sigma` represents the precision parameter  $\delta$ ) has density given by

$$f(x|\mu, \sigma) = \frac{\exp(\sigma/2)\sqrt{\sigma+1}}{4\sqrt{\pi}\mu x^{3/2}} \left[ x + \frac{\sigma\mu}{\sigma+1} \right] \exp\left(-\frac{\sigma}{4} \left[ \frac{x(\sigma+1)}{\sigma\mu} + \frac{\sigma\mu}{x(\sigma+1)} \right]\right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \mu$  and  $Var(X) = \mu^2 \left[ \frac{2\sigma+5}{(\sigma+1)^2} \right]$ .

### Value

Returns a `gamlss.family` object which can be used to fit a BS5 distribution in the `gamlss()` function.

### Author(s)

David Villegas Ceballos, <david.villegas1@udea.edu.co>

### References

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

### See Also

[dBS5](#).

## Examples

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(123)
y <- rBS5(n=50, mu=1, sigma=25)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS5)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y ~ BS5
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(1.5 - 3 * x1)      # Aprox 1
  sigma <- exp(2.4 + 1.7 * x2) # Aprox 25
  y <- rBS5(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

dat <- gendat(n=100)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
              family=BS5, data=dat)

summary(mod2)
```

---

 BS6

*The Birnbaum-Saunders family - Santos-Neto et al. (2012) (P4 Based on the mean)*

---

## Description

The function `BS6()` defines the Birnbaum-Saunders distribution, a two-parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

## Usage

```
BS6(mu.link = "log", sigma.link = "log")
```

**Arguments**

mu.link defines the mu.link, with "log" link as the default for the mu parameter.  
 sigma.link defines the sigma.link, with "log" link as the default for the sigma parameter.

**Details**

The Birnbaum-Saunders distribution with parameters mu and sigma (where mu represents the true mean and sigma represents the shape parameter  $\alpha$ ) has density given by

$$f(x|\mu, \sigma) = \frac{\exp(1/\sigma^2)\sqrt{2+\sigma^2}}{4\sigma\sqrt{\pi}\mu x^{3/2}} \left[ x + \frac{2\mu}{2+\sigma^2} \right] \exp\left(-\frac{1}{2\sigma^2} \left[ \frac{\{2+\sigma^2\}x}{2\mu} + \frac{2\mu}{\{2+\sigma^2\}x} \right]\right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \mu$  and  $Var(X) = [\mu\sigma]^2 \left[ \frac{4+5\sigma^2}{(2+\sigma^2)^2} \right]$ .

**Value**

Returns a gamlss.family object which can be used to fit a BS6 distribution in the gamlss() function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. Pakistan Journal of Statistics, 28(1), 1-26.

**See Also**

[dBS6](#).

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(1234)
y <- rBS6(n=50, mu=1, sigma=0.1)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS6)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model
```

```

# A function to simulate a data set with Y ~ BS6
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(1.5 - 3 * x1)      # Aprox 1
  sigma <- exp(0.5 - 3.5 * x2) # Aprox 0.1
  y <- rBS6(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

dat <- gendat(n=100)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
               family=BS6, data=dat)

summary(mod2)

```

BS7

*The Birnbaum-Saunders family - Santos-Neto et al. (2012) (P5 Based on the variance)*

## Description

The function `BS7()` defines the Birnbaum-Saunders distribution, a two-parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

## Usage

```
BS7(mu.link = "log", sigma.link = "log")
```

## Arguments

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter (representing the variance).

`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma` parameter (representing the shape).

## Details

The Birnbaum-Saunders distribution with parameters `mu` and `sigma` (where `mu` represents the true variance  $\sigma^2$  and `sigma` represents the shape parameter  $\alpha$ ) has density given by

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2\mu^2} \left[ \frac{\mu\sqrt{4+5\mu^2}}{2\sqrt{\sigma}x^{-1}} + \frac{2\sqrt{\sigma}\{x\mu\}^{-1}}{\sqrt{4+5\mu^2}} - 2 \right]\right) \times \left[ \frac{\{x\mu\}^{-1/2}\{4+5\mu^2\}^{1/4}}{2^{3/2}\sigma^{1/4}} + \frac{\sigma^{1/4}}{\{x\mu\}^{3/2}\sqrt{2}\{4+5\mu^2\}^{1/4}} \right]$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \frac{[2+\mu^2]\sqrt{\sigma}}{\mu\sqrt{4+5\mu^2}}$  and  $Var(X) = \sigma$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a BS7 distribution in the `gamlss()` function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

**See Also**

[dBS7](#).

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(12345)
y <- rBS7(n=100, mu=0.2, sigma=10)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=BS7,
               control=gamlss.control(n.cyc=1000))

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y ~ BS7
## Not run:
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(0.6 - 4.4 * x1)      # Aprox 0.2
  sigma <- exp(1.6 + 1.5 * x2)  # Aprox 10
  y <- rBS7(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(123)
dat <- gendat(n=200)
```

```
mod2 <- gamlss(y~x1, sigma.fo=~x2,
              family=BS7, data=dat,
              control=gamlss.control(n.cyc=1000))

summary(mod2)

## End(Not run)
```

CJ2

*The two-parameter Chris-Jerry distribution family***Description**

The function `CJ2()` defines The two-parameter Chris-Jerry distribution, a two parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

**Usage**

```
CJ2(mu.link = "log", sigma.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.

**Details**

The two-parameter Chris-Jerry distribution with parameters `mu` and `sigma` has density given by

$$f(x; \sigma, \mu) = \frac{\mu^2}{\sigma\mu+2}(\sigma + \mu x^2)e^{-\mu x}; \quad x > 0, \quad \mu > 0, \quad \sigma > 0$$

Note: In this implementation we changed the original parameters  $\theta$  for  $\mu$  and  $\lambda$  for  $\sigma$  we did it to implement this distribution within `gamlss` framework.

**Value**

Returns a `gamlss.family` object which can be used to fit a CJ2 distribution in the `gamlss()` function.

**Author(s)**

Manuel Gutierrez Tangarife, <mgutierrez@unal.edu.co>

**References**

Chinedu, Eberechukwu Q., et al. "New lifetime distribution with applications to single acceptance sampling plan and scenarios of increasing hazard rates" *Symmetry* 15.10 (2023): 188.

**See Also**

[dCJ2](#)

**Examples**

```

# Example 1
# Generating some random values with
# known mu and sigma
y <- rCJ2(n=500, mu=1, sigma=1.5)

# Fitting the model
require(gamlss)

mod1 <- gamlss(y~1, sigma.fo=~1, family=CJ2,
               control=gamlss.control(n.cyc=5000, trace=TRUE))

# Extracting the fitted values for mu, sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values under some model
gendat <- function(n) {
  x1 <- runif(n, min=0, max=5)
  x2 <- runif(n, min=0, max=5)
  mu <- exp(-0.2 + 1.5 * x1)
  sigma <- exp(1 - 0.7 * x2)
  y <- rCJ2(n=n, mu, sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(123)
datos <- gendat(n=500)

mod2 <- gamlss(y~x1, sigma.fo=~x2, family=CJ2, data=datos,
               control=gamlss.control(n.cyc=5000, trace=TRUE))

summary(mod2)

```

**Description**

The Cosine Sine Exponential family

**Usage**

```
CS2e(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the <code>mu</code> parameter.
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the <code>sigma</code> .
<code>nu.link</code>	defines the <code>nu.link</code> , with "log" link as the default for the <code>nu</code> parameter.

**Details**

The Cosine Sine Exponential distribution with parameters `mu`, `sigma` and `nu` has density given by

$$f(x) = \frac{\pi \sigma \mu \exp\left(\frac{-x}{\nu}\right)}{2\nu[(\mu \sin\left(\frac{\pi}{2} \exp\left(\frac{-x}{\nu}\right)\right) + \sigma \cos\left(\frac{\pi}{2} \exp\left(\frac{-x}{\nu}\right)\right)]^2},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a CS2e distribution in the `gamlss()` function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

Chesneau, C., Bakouch, H. S., & Hussain, T. (2019). A new class of probability distributions via cosine and sine functions with applications. *Communications in Statistics-Simulation and Computation*, 48(8), 2287-2300.

**See Also**

[dCS2e](#)

**Examples**

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rCS2e(n=100, mu = 0.1, sigma =1, nu=0.5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='CS2e',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))
```

```

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n, min=0.45, max=0.55)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(0.2 - x1)
sigma <- exp(0.8 - x2)
nu <- 0.5
x <- rCS2e(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, family=CS2e,
              control=gamlss.control(n.cyc=50000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))

```

dAddW

*The Additive Weibull distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Additive Weibull distribution with parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$ .

**Usage**

```

dAddW(x, mu, sigma, nu, tau, log = FALSE)

pAddW(q, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)

qAddW(p, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)

rAddW(n, mu, sigma, nu, tau)

hAddW(x, mu, sigma, nu, tau)

```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter.
<code>sigma</code>	parameter.
<code>nu</code>	shape parameter.
<code>tau</code>	shape parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

## Details

Additive Weibull Distribution with parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$  has density given by

$$f(x) = (\mu\nu x^{\nu-1} + \sigma\tau x^{\tau-1}) \exp(-\mu x^{\nu} - \sigma x^{\tau}),$$

for  $x > 0$ .

## Value

dAddW gives the density, pAddW gives the distribution function, qAddW gives the quantile function, rAddW generates random deviates and hAddW gives the hazard function.

## Author(s)

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

## References

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Xie, M., & Lai, C. D. (1996). Reliability analysis using an additive Weibull model with bathtub-shaped failure rate function. *Reliability Engineering & System Safety*, 52(1), 87-93.

## See Also

[AddW](#)

## Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dAddW(x, mu=1.5, sigma=0.5, nu=3, tau=0.8), from=0.0001, to=2,
      col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pAddW(x, mu=1.5, sigma=0.5, nu=3, tau=0.8),
      from=0.0001, to=2, col="red", las=1, ylab="F(x)")
curve(pAddW(x, mu=1.5, sigma=0.5, nu=3, tau=0.8, lower.tail=FALSE),
      from=0.0001, to=2, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qAddW(p, mu=1.5, sigma=0.2, nu=3, tau=0.8), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pAddW(x, mu=1.5, sigma=0.2, nu=3, tau=0.8),
      from=0, add=TRUE, col="red")

## The random function
hist(rAddW(n=10000, mu=1.5, sigma=0.2, nu=3, tau=0.8), freq=FALSE,
     xlab="x", las=1, main="")
```

```

curve(dAddW(x, mu=1.5, sigma=0.2, nu=3, tau=0.8),
      from=0.09, to=5, add=TRUE, col="red")

## The Hazard function
curve(hAddW(x, mu=1.5, sigma=0.2, nu=3, tau=0.8), from=0.001, to=1,
      col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters

```

dBGE

*The Beta Generalized Exponentiated distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Beta Generalized Exponentiated distribution with parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$ .

**Usage**

```

dBGE(x, mu, sigma, nu, tau, log = FALSE)

pBGE(q, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)

qBGE(p, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)

rBGE(n, mu, sigma, nu, tau)

hBGE(x, mu, sigma, nu, tau)

```

**Arguments**

$x, q$	vector of quantiles.
$\mu$	parameter.
$\sigma$	parameter.
$\nu$	parameter.
$\tau$	parameter.
$\log, \log.p$	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
$\text{lower.tail}$	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
$p$	vector of probabilities.
$n$	number of observations.

**Details**

The Beta Generalized Exponentiated Distribution with parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$  has density given by

$$f(x) = \frac{\nu\tau}{B(\mu, \sigma)} \exp(-\nu x) (1 - \exp(-\nu x))^{\tau\mu-1} (1 - (1 - \exp(-\nu x))^{\tau})^{\sigma-1},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$ ,  $\nu > 0$  and  $\tau > 0$ .

**Value**

dBGE gives the density, pBGE gives the distribution function, qBGE gives the quantile function, rBGE generates random deviates and hBGE gives the hazard function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

- Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.
- Barreto-Souza, W., Santos, A. H., & Cordeiro, G. M. (2010). The beta generalized exponential distribution. *Journal of statistical Computation and Simulation*, 80(2), 159-172.

**See Also**

[BGE](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dBGE(x, mu = 1.5, sigma =1.7, nu=1, tau=1), from = 0, to = 3,
      col = "red", las = 1, ylab = "f(x)")

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pBGE(x, mu = 1.5, sigma =1.7, nu=1, tau=1), from = 0, to = 6,
      ylim = c(0, 1), col = "red", las = 1, ylab = "F(x)")
curve(pBGE(x, mu = 1.5, sigma =1.7, nu=1, tau=1, lower.tail = FALSE),
      from = 0, to = 6, ylim = c(0, 1), col = "red", las = 1, ylab = "R(x)")

## The quantile function
p <- seq(from = 0, to = 0.99999, length.out = 100)
plot(x = qBGE(p = p, mu = 1.5, sigma =1.7, nu=1, tau=1), y = p,
     xlab = "Quantile", las = 1, ylab = "Probability")
curve(pBGE(x, mu = (1/4), sigma =1, nu=1, tau=2), from = 0, add = TRUE,
     col = "red")

## The random function
hist(rBGE(1000, mu = 1.5, sigma =1.7, nu=1, tau=1), freq = FALSE, xlab = "x",
     ylim = c(0, 1), las = 1, main = "")
curve(dBGE(x, mu = 1.5, sigma =1.7, nu=1, tau=1), from = 0, add = TRUE,
     col = "red", ylim = c(0, 0.5))

## The Hazard function(
par(mfrow=c(1,1))
curve(hBGE(x, mu = 0.9, sigma =0.5, nu=1, tau=1), from = 0, to = 2,
     col = "red", ylab = "Hazard function", las = 1)
```

```
par(old_par) # restore previous graphical parameters
```

---

dBS

*The Birnbaum-Saunders distribution*


---

### Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

### Usage

```
dBS(x, mu = 1, sigma = 1, log = FALSE)
pBS(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qBS(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rBS(n, mu = 1, sigma = 1)
hBS(x, mu, sigma)
```

### Arguments

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter.
<code>sigma</code>	parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

### Details

The Birnbaum-Saunders with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x|\mu, \sigma) = \frac{x^{-3/2}(x+\mu)}{2\sigma\sqrt{2\pi\mu}} \exp\left(\frac{-1}{2\sigma^2}\left(\frac{x}{\mu} + \frac{\mu}{x} - 2\right)\right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization  $\mu$  is the median of  $X$ ,  $E(X) = \mu(1 + \sigma^2/2)$  and  $Var(X) = (\mu\sigma)^2(1 + 5\sigma^2/4)$ . The functions proposed here corresponds to the functions created by Roquim et al. (2021) with minor modifications to obtain correct log-likelihoods and random samples.

### Value

dBS gives the density, pBS gives the distribution function, qBS gives the quantile function, rBS generates random deviates and hBS gives the hazard function.

## References

Birnbaum, Z.W. and Saunders, S.C. (1969a). A new family of life distributions. *J. Appl. Prob.*, 6, 319–327.

Roquim, F. V., Ramires, T. G., Nakamura, L. R., Righetto, A. J., Lima, R. R., & Gomes, R. A. (2021). Building flexible regression models: including the Birnbaum-Saunders distribution in the *gamlss* package. *Semina: Ciências Exatas e Tecnológicas*, 42(2), 163-168.

## See Also

[BS](#).

## Examples

```
# Example 1
# Plotting the mass function for different parameter values
curve(dBS(x, mu=0.5, sigma=0.5),
      from=0.001, to=5,
      ylim=c(0, 2),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS(x, mu=1, sigma=0.5),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(dBS(x, mu=1.5, sigma=0.5),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=0.5, sigma=0.5",
                           "mu=1.0, sigma=0.5",
                           "mu=1.5, sigma=0.5"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.6)

curve(dBS(x, mu=0.5, sigma=1),
      from=0.001, to=5,
      ylim=c(0, 1.5),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS(x, mu=1, sigma=1),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(dBS(x, mu=1.5, sigma=1),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=0.5, sigma=1",
                           "mu=1.0, sigma=1",
```

```

                                "mu=1.5, sigma=1"),
                                col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.6)

curve(dBS(x, mu=0.5, sigma=1.5),
      from=0.001, to=8,
      ylim=c(0, 2),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS(x, mu=1, sigma=1.5),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(dBS(x, mu=1.5, sigma=1.5),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=0.5, sigma=1.5",
                            "mu=1.0, sigma=1.5",
                            "mu=1.5, sigma=1.5"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS(x, mu=0.5, sigma=0.5),
      from=0.001, to=5,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS(x, mu=1, sigma=0.5),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(pBS(x, mu=1.5, sigma=0.5),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=0.5, sigma=0.5",
                              "mu=1.0, sigma=0.5",
                              "mu=1.5, sigma=0.5"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.5)
curve(pBS(x, mu=0.5, sigma=0.5, lower.tail=FALSE),
      from=0.001, to=5,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS(x, mu=1, sigma=0.5, lower.tail=FALSE),
      col="tomato",
      lwd=2,
      add=TRUE)

```

```

curve(pBS(x, mu=1.5, sigma=0.5, lower.tail=FALSE),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=0.5, sigma=0.5",
                            "mu=1.0, sigma=0.5",
                            "mu=1.5, sigma=0.5"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS(p, mu=2.3, sigma=1.7), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pBS(x, mu=2.3, sigma=1.7),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rBS(n=10000, mu=20, sigma=0.5)
hist(x, freq=FALSE)
curve(dBS(x, mu=20, sigma=0.5), from=0, to=100,
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hBS(x, mu=20, sigma=0.5), from=0.001, to=100,
      col="tomato", ylab="Hazard function", las=1)

```

---

dBS10

*The Birnbaum-Saunders distribution - Santos-Neto et al. (2012) (P8  
Based on the first Tweedie)*

---

## Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

## Usage

```

dBS10(x, mu = 1, sigma = 0.5, log = FALSE)

pBS10(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

qBS10(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

rBS10(n, mu = 1, sigma = 0.5)

hBS10(x, mu, sigma)

```

**Arguments**

x, q	vector of quantiles.
mu	parameter representing $\tau$ ( $\mu > 0$ ).
sigma	parameter representing $\omega$ ( $\sigma > 0$ ).
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

**Details**

The Birnbaum-Saunders with parameters mu and sigma has density given by

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\sigma}{\mu^2} \left[\frac{2x}{\mu^2} + \frac{\mu^2}{2x} - 2\right]\right) \frac{[2x + \mu^2]\sqrt{\sigma}}{2\mu^2\sqrt{x^3}}$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \frac{\mu^2}{2} + \frac{\mu^4}{8\sigma}$  and  $Var(X) = \frac{\mu^6}{8\sigma} + \frac{5\mu^8}{64\sigma^2}$ .

**Value**

dBS10 gives the density, pBS10 gives the distribution function, qBS10 gives the quantile function, rBS10 generates random deviates and hBS10 gives the hazard function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

**See Also**

[BS10](#).

**Examples**

```
# Example 1
# Plotting the mass function for different parameter values
curve(dBS10(x, mu=0.75, sigma=5),
      from=0.001, to=1.5,
      ylim=c(0, 6.2),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS10(x, mu=1.15, sigma=5),
      col="tomato",
      lwd=2,
      add=TRUE)
```

```

legend("topright", legend=c("mu=0.75, sigma=5",
                             "mu=1.15, sigma=5"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS10(x, mu=0.75, sigma=5),
      from=0.00001, to=1.5,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS10(x, mu=1.15, sigma=5),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=0.75, sigma=5",
                                "mu=1.15, sigma=5"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS10(p, mu=0.75, sigma=5), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pBS10(x, mu=0.75, sigma=5),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rBS10(n=10000, mu=0.75, sigma=5)
hist(x, freq=FALSE)
curve(dBS10(x, mu=0.75, sigma=5),
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hBS10(x, mu=0.75, sigma=5), from=0.001, to=2,
      col="tomato", ylab="Hazard function", las=1)

```

### Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

**Usage**

```
dBS11(x, mu = 1, sigma = 0.5, log = FALSE)

pBS11(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

qBS11(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

rBS11(n, mu = 1, sigma = 0.5)

hBS11(x, mu, sigma)
```

**Arguments**

x, q	vector of quantiles.
mu	parameter representing $\beta$ ( $\mu > 0$ ).
sigma	parameter representing $\omega$ ( $\sigma > 0$ ).
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

**Details**

The Birnbaum-Saunders with parameters mu and sigma has density given by

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\sigma}{2\mu} \left[\frac{x}{\mu} + \frac{\mu}{x} - 2\right]\right) \frac{[x+\mu]\sqrt{\sigma}}{2\mu\sqrt{x^3}}$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \mu + \frac{\mu^2}{2\sigma}$  and  $Var(X) = \frac{\mu^3}{\sigma} + \frac{5\mu^4}{4\sigma^2}$ .

**Value**

dBS11 gives the density, pBS11 gives the distribution function, qBS11 gives the quantile function, rBS11 generates random deviates and hBS11 gives the hazard function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

**See Also**

[BS11](#).

**Examples**

```

# Example 1
# Plotting the mass function for different parameter values
curve(dBS11(x, mu=1, sigma=12),
      from=0.001, to=2.5,
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS11(x, mu=1, sigma=0.5),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=1, sigma=12",
                           "mu=1, sigma=0.5"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS11(x, mu=1, sigma=12),
      from=0.00001, to=8,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS11(x, mu=1, sigma=0.5),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=1, sigma=12",
                              "mu=1, sigma=0.5"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS11(p, mu=1, sigma=12), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pBS11(x, mu=1, sigma=12),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rBS11(n=10000, mu=1, sigma=12)
hist(x, freq=FALSE)
curve(dBS11(x, mu=1, sigma=12),
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hBS11(x, mu=1, sigma=12), from=0.001, to=4,
      col="tomato", ylab="Hazard function", las=1)

```

---

dBS12 *The Birnbaum-Saunders distribution - Santos-Neto et al. (2012) (P10 Based on the third Tweedie)*

---

### Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

### Usage

```
dBS12(x, mu = 1, sigma = 0.5, log = FALSE)
```

```
pBS12(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
```

```
qBS12(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
```

```
rBS12(n, mu = 1, sigma = 0.5)
```

```
hBS12(x, mu, sigma)
```

### Arguments

x, q	vector of quantiles.
mu	parameter representing $\beta$ ( $\mu > 0$ ).
sigma	parameter representing $\psi$ ( $\sigma > 0$ ).
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

### Details

The Birnbaum-Saunders with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\sigma}{2} \left[\frac{x}{\mu} + \frac{\mu}{x} - 2\right]\right) \frac{[x+\mu]\sqrt{\sigma}}{2\sqrt{\mu x^3}}$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \mu + \frac{\mu}{2\sigma}$  and  $Var(X) = \frac{\mu^2}{\sigma} + \frac{5\mu^2}{4\sigma^2}$ .

### Value

dBS12 gives the density, pBS12 gives the distribution function, qBS12 gives the quantile function, rBS12 generates random deviates and hBS12 gives the hazard function.

### Author(s)

David Villegas Ceballos, <david.villegas1@udea.edu.co>

## References

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

## See Also

[BS12](#).

## Examples

```
# Example 1
# Plotting the mass function for different parameter values
curve(dBS12(x, mu=1, sigma=30),
      from=0.001, to=2.5,
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS12(x, mu=1, sigma=0.5),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=1, sigma=30",
                            "mu=1, sigma=0.5"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS12(x, mu=1, sigma=30),
      from=0.00001, to=8,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS12(x, mu=1, sigma=0.5),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=1, sigma=30",
                              "mu=1, sigma=0.5"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS12(p, mu=1, sigma=30), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pBS12(x, mu=1, sigma=30),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rBS12(n=10000, mu=1, sigma=30)
```

```

hist(x, freq=FALSE)
curve(dBS12(x, mu=1, sigma=30),
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hBS12(x, mu=1, sigma=30), from=0.001, to=4,
      col="tomato", ylab="Hazard function", las=1)

```

---

dBS13	<i>The Birnbaum-Saunders distribution - Santos-Neto et al. (2012) (P11 Based on the fourth Tweedie)</i>
-------	---

---

### Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

### Usage

```

dBS13(x, mu = 1, sigma = 0.5, log = FALSE)

pBS13(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

qBS13(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

rBS13(n, mu = 1, sigma = 0.5)

hBS13(x, mu, sigma)

```

### Arguments

x, q	vector of quantiles.
mu	parameter representing $\omega$ ( $\mu > 0$ ).
sigma	parameter representing $\psi$ ( $\sigma > 0$ ).
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

### Details

The Birnbaum-Saunders with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\sigma}{2} \left[\frac{x\sigma}{\mu} + \frac{\mu}{x\sigma} - 2\right]\right) \frac{[x\sigma + \mu]}{2\sqrt{\mu x^3}}$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \frac{\mu}{\sigma} + \frac{\mu}{2\sigma^2}$  and  $Var(X) = \frac{\mu^2}{\sigma^3} + \frac{5\mu^2}{4\sigma^4}$ .

**Value**

dBS13 gives the density, pBS13 gives the distribution function, qBS13 gives the quantile function, rBS13 generates random deviates and hBS13 gives the hazard function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

**See Also**

[BS13](#).

**Examples**

```
# Example 1
# Plotting the mass function for different parameter values
curve(dBS13(x, mu=5, sigma=30),
      from=0.001, to=0.8,
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS13(x, mu=5, sigma=10),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=5, sigma=30",
                           "mu=5, sigma=10"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS13(x, mu=5, sigma=30),
      from=0.00001, to=2,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS13(x, mu=5, sigma=10),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=5, sigma=30",
                              "mu=5, sigma=10"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.5)

# Example 3
# The quantile function
```

```

p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS13(p, mu=5, sigma=30), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pBS13(x, mu=5, sigma=30),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rBS13(n=10000, mu=5, sigma=30)
hist(x, freq=FALSE)
curve(dBS13(x, mu=5, sigma=30),
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hBS13(x, mu=5, sigma=30), from=0.001, to=1,
      col="tomato", ylab="Hazard function", las=1)

```

---

dBS2

*The Birnbaum-Saunders distribution - Santos-Neto et al. (2014)*


---

## Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

## Usage

```

dBS2(x, mu = 1, sigma = 1, log = FALSE)

pBS2(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)

qBS2(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)

rBS2(n, mu = 1, sigma = 1)

hBS2(x, mu, sigma)

```

## Arguments

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter.
<code>sigma</code>	parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

## Details

The Birnbaum-Saunders with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x) = \frac{\exp(\sigma/2)\sqrt{\sigma+1}}{4\sqrt{\pi}\mu x^{3/2}} \left[ x + \frac{\mu\sigma}{\sigma+1} \right] \exp\left(\frac{-\sigma}{4} \left( \frac{x(\sigma+1)}{\mu\sigma} + \frac{\mu\sigma}{x(\sigma+1)} \right)\right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization  $E(X) = \mu$  and  $Var(X) = (\mu\sigma)^2(1+5\sigma^2/4)$ . The functions proposed here corresponds to the parameterization proposed by Santos-Neto et al. (2014).

## Value

dBS2 gives the density, pBS2 gives the distribution function, qBS2 gives the quantile function, rBS2 generates random deviates and hBS2 gives the hazard function.

## References

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Barros, M. (2014). A reparameterized Birnbaum-Saunders distribution and its moments, estimation and applications. *REVSTAT-Statistical Journal*, 12(3), 247-272.

## See Also

[BS2](#).

## Examples

```
#Example 1
#Plotting the mass function for different parameter values
curve(dBS2(x, mu=1.0, sigma=100),
      from=0.001, to=5,
      ylim=c(0, 3),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS2(x, mu=1.5, sigma=100),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(dBS2(x, mu=2.0, sigma=100),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=1.0, sigma=100",
                            "mu=1.5, sigma=100",
                            "mu=2.0, sigma=100"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.6)

curve(dBS2(x, mu=1, sigma=2),
      from=0.001, to=2,
      ylim=c(0, 1.1),
      col="royalblue1", lwd=2,
```

```

    main="Density function",
    xlab="x", ylab="f(x)")
curve(dBS2(x, mu=1, sigma=5),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(dBS2(x, mu=1, sigma=10),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=1, sigma=2",
                            "mu=1, sigma=5",
                            "mu=1, sigma=10"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.6)

```

```

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS2(x, mu=0.5, sigma=0.5),
      from=0.001, to=15,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS2(x, mu=1, sigma=0.5),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(pBS2(x, mu=1.5, sigma=0.5),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=0.5, sigma=0.5",
                              "mu=1.0, sigma=0.5",
                              "mu=1.5, sigma=0.5"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.5)

```

```

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS2(p, mu=2.3, sigma=1.7), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pBS2(x, mu=2.3, sigma=1.7),
      from=0, add=TRUE, col="tomato", lwd=2.5)

```

```

# Example 4
# The random function
x <- rBS2(n=10000, mu=2.5, sigma=100)
hist(x, freq=FALSE)
curve(dBS2(x, mu=2.5, sigma=100), from=0, to=10,
      add=TRUE, col="tomato", lwd=2)

```

```

# Example 5

```

```
# The Hazard function
curve(hBS2(x, mu=20, sigma=0.5), from=0.001, to=100,
      col="tomato", ylab="Hazard function", las=1)
```

---

dBS3 *The Birnbaum-Saunders distribution - Bourguignon & Gallardo (2022)*

---

### Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

### Usage

```
dBS3(x, mu = 1, sigma = 0.5, log = FALSE)

pBS3(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

qBS3(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

rBS3(n, mu = 1, sigma = 0.5)

hBS3(x, mu, sigma)
```

### Arguments

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter.
<code>sigma</code>	parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities p are given as log(p).
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

### Details

The Birnbaum-Saunders with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x) = \frac{\exp(\sigma/2)\sqrt{\sigma+1}}{4\sqrt{\pi\mu x^{3/2}}} \left[ x + \frac{\mu\sigma}{\sigma+1} \right] \exp\left(\frac{-\sigma}{4} \left( \frac{x(\sigma+1)}{\mu\sigma} + \frac{\mu\sigma}{x(\sigma+1)} \right)\right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization  $E(X) = \mu$  and  $Var(X) = (\mu\sigma)^2(1+5\sigma^2/4)$ . The functions proposed here corresponds to the parameterization proposed by Santos-Neto et al. (2014).

**Value**

dBS3 gives the density, pBS3 gives the distribution function, qBS3 gives the quantile function, rBS3 generates random deviates and hBS3 gives the hazard function.

**References**

Bourguignon, M., & Gallardo, D. I. (2022). A new look at the Birnbaum–Saunders regression model. *Applied Stochastic Models in Business and Industry*, 38(6), 935-951.

**See Also**

[BS3](#).

**Examples**

```
# Example 1
# Plotting the mass function for different parameter values
curve(dBS3(x, mu=2, sigma=0.2),
      from=0.001, to=10,
      ylim=c(0, 0.4),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS3(x, mu=2, sigma=0.4),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=2, sigma=0.2",
                           "mu=2, sigma=0.4"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS3(x, mu=2, sigma=0.2),
      from=0.00001, to=10,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS3(x, mu=2, sigma=0.4),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=2, sigma=0.2",
                              "mu=2, sigma=0.4"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS3(p, mu=2, sigma=0.2), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
```

```

curve(pBS3(x, mu=2, sigma=0.2),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rBS3(n=10000, mu=2, sigma=0.2)
hist(x, freq=FALSE)
curve(dBS3(x, mu=2, sigma=0.2), from=0, to=10,
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hBS3(x, mu=2, sigma=0.2), from=0.001, to=4,
      col="tomato", ylab="Hazard function", las=1)

```

---

dBS4

*The Birnbaum-Saunders distribution - Ahmed et al. (2008)*


---

## Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

## Usage

```

dBS4(x, mu = 1, sigma = 0.5, log = FALSE)

pBS4(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

qBS4(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

rBS4(n, mu = 1, sigma = 0.5)

hBS4(x, mu, sigma)

```

## Arguments

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter ( $\mu > 0$ ).
<code>sigma</code>	parameter ( $\sigma > 0$ ).
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

**Details**

The Birnbaum-Saunders with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x|\mu, \sigma) = \frac{1}{2\sqrt{2\pi}} \left[ \frac{\sigma}{x\sqrt{x}} + \frac{\mu}{\sqrt{x}} \right] \exp \left( -\frac{1}{2} \left[ \frac{\sigma}{\sqrt{x}} - \mu\sqrt{x} \right]^2 \right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization  $E(X) = \frac{\sigma\mu+1/2}{\mu^2}$  and  $Var(X) = \frac{\sigma\mu+5/4}{\mu^4}$ .

**Value**

dBS4 gives the density, pBS4 gives the distribution function, qBS4 gives the quantile function, rBS4 generates random deviates and hBS4 gives the hazard function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Ahmed, S. E., Budsaba, K., Lisawadi, S., & Volodin, A. (2008). Parametric estimation for the Birnbaum-Saunders lifetime distribution based on a new parametrization. *Thailand Statistician*, 6(2), 213-240.

**See Also**

[BS4](#).

**Examples**

```
# Example 1
# Plotting the mass function for different parameter values
curve(dBS4(x, mu=2, sigma=30),
      from=0.001, to=40,
      ylim=c(0, 0.20),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS4(x, mu=1, sigma=20),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=2, sigma=30",
                            "mu=1, sigma=20"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS4(x, mu=2, sigma=30),
      from=0.00001, to=40,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
```

```

      xlab="x", ylab="F(x)")
curve(pBS4(x, mu=1, sigma=20),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=2, sigma=30",
                               "mu=1, sigma=20"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS4(p, mu=2, sigma=30), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pBS4(x, mu=2, sigma=30),
     from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rBS4(n=10000, mu=2, sigma=30)
hist(x, freq=FALSE)
curve(dBS4(x, mu=2, sigma=30), from=0, to=30,
     add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hBS4(x, mu=2, sigma=30), from=0.001, to=40,
     col="tomato", ylab="Hazard function", las=1)

```

---

dBS5

*The Birnbaum-Saunders distribution - Santos-Neto et al. (2012) (P3  
Based on GLM)*

---

### Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

### Usage

```

dBS5(x, mu = 1, sigma = 0.5, log = FALSE)

pBS5(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

qBS5(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

rBS5(n, mu = 1, sigma = 0.5)

hBS5(x, mu, sigma)

```

**Arguments**

x, q	vector of quantiles.
mu	parameter ( $\mu > 0$ ).
sigma	precision parameter $\delta$ ( $\sigma > 0$ ).
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

**Details**

The Birnbaum-Saunders with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x|\mu, \sigma) = \frac{\exp(\frac{\sigma}{2})\sqrt{\sigma+1}}{4\sqrt{\pi}\mu x^{3/2}} \left[ x + \frac{\sigma\mu}{\sigma+1} \right] \exp\left(-\frac{\sigma}{4} \left[ \frac{x(\sigma+1)}{\sigma\mu} + \frac{\sigma\mu}{x(\sigma+1)} \right]\right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization  $E(X) = \mu$  and  $Var(X) = \mu^2 \left[ \frac{2\sigma+5}{(\sigma+1)^2} \right]$ .

**Value**

dBS5 gives the density, pBS5 gives the distribution function, qBS5 gives the quantile function, rBS5 generates random deviates and hBS5 gives the hazard function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

**See Also**

[BS5](#).

**Examples**

```
# Example 1
# Plotting the mass function for different parameter values
curve(dBS5(x, mu=1, sigma=2),
      from=0.001, to=2,
      ylim=c(0, 1.5),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS5(x, mu=1, sigma=25),
      col="tomato",
      lwd=2,
      add=TRUE)
```

```

legend("topright", legend=c("mu=1, sigma=2",
                            "mu=1, sigma=25"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS5(x, mu=1, sigma=2),
      from=0.00001, to=6,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS5(x, mu=1, sigma=25),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=1, sigma=2",
                              "mu=1, sigma=25"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS5(p, mu=1, sigma=2), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pBS5(x, mu=1, sigma=2),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rBS5(n=10000, mu=1, sigma=25)
hist(x, freq=FALSE)
curve(dBS5(x, mu=1, sigma=25),
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hBS5(x, mu=1, sigma=25), from=0.001, to=6,
      col="tomato", ylab="Hazard function", las=1)

```

### Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

**Usage**

```
dBS6(x, mu = 1, sigma = 0.5, log = FALSE)
pBS6(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
qBS6(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
rBS6(n, mu = 1, sigma = 0.5)
hBS6(x, mu, sigma)
```

**Arguments**

x, q	vector of quantiles.
mu	parameter representing the mean ( $\mu > 0$ ).
sigma	parameter representing the shape $\alpha$ ( $\sigma > 0$ ).
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

**Details**

The Birnbaum-Saunders with parameters mu and sigma has density given by

$$f(x|\mu, \sigma) = \frac{\exp(1/\sigma^2)\sqrt{2+\sigma^2}}{4\sigma\sqrt{\pi}\mu x^{3/2}} \left[ x + \frac{2\mu}{2+\sigma^2} \right] \exp\left(-\frac{1}{2\sigma^2} \left[ \frac{\{2+\sigma^2\}x}{2\mu} + \frac{2\mu}{\{2+\sigma^2\}x} \right]\right)$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \mu$  and  $Var(X) = [\mu\sigma]^2 \left[ \frac{4+5\sigma^2}{(2+\sigma^2)^2} \right]$ .

**Value**

dBS6 gives the density, pBS6 gives the distribution function, qBS6 gives the quantile function, rBS6 generates random deviates and hBS6 gives the hazard function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

**See Also**

[BS6](#).

**Examples**

```

# Example 1
# Plotting the mass function for different parameter values
curve(dBS6(x, mu=2, sigma=0.1),
      from=0.001, to=3,
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS6(x, mu=2, sigma=0.75),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=2, sigma=0.1",
                           "mu=2, sigma=0.75"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS6(x, mu=2, sigma=0.1),
      from=0.00001, to=6,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS6(x, mu=2, sigma=0.75),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=2, sigma=0.1",
                              "mu=2, sigma=0.75"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS6(p, mu=1, sigma=2), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pBS6(x, mu=1, sigma=2),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rBS6(n=10000, mu=2, sigma=0.1)
hist(x, freq=FALSE)
curve(dBS6(x, mu=2, sigma=0.1),
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hBS6(x, mu=2, sigma=0.1), from=0.001, to=6,
      col="tomato", ylab="Hazard function", las=1)

```

---

dBS7	<i>The Birnbaum-Saunders distribution - Santos-Neto et al. (2012) (P5 Based on the variance)</i>
------	--

---

### Description

Density, distribution function, quantile function, random generation and hazard function for the Birnbaum-Saunders distribution with parameters  $\mu$  and  $\sigma$ .

### Usage

dBS7(x, mu = 0.5, sigma = 10, log = FALSE)

pBS7(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

qBS7(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)

rBS7(n, mu = 1, sigma = 0.5)

hBS7(x, mu, sigma)

### Arguments

x, q	vector of quantiles.
mu	parameter representing the shape ( $\mu > 0$ ).
sigma	parameter representing the variance ( $\sigma > 0$ ).
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

### Details

The Birnbaum-Saunders with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2\mu^2} \left[ \frac{\mu\sqrt{4+5\mu^2}}{2\sqrt{\sigma}x^{-1}} + \frac{2\sqrt{\sigma}\{x\mu\}^{-1}}{\sqrt{4+5\mu^2}} - 2 \right]\right) \times \left[ \frac{\{x\mu\}^{-1/2}\{4+5\mu^2\}^{1/4}}{2^{3/2}\sigma^{1/4}} + \frac{\sigma^{1/4}}{\{x\mu\}^{3/2}\sqrt{2}\{4+5\mu^2\}^{1/4}} \right]$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ . In this parameterization,  $E(X) = \frac{[2+\mu^2]\sqrt{\sigma}}{\mu\sqrt{4+5\mu^2}}$  and  $Var(X) = \sigma$ .

### Value

dBS7 gives the density, pBS7 gives the distribution function, qBS7 gives the quantile function, rBS7 generates random deviates and hBS7 gives the hazard function.

**Author(s)**

David Villegas Ceballos, <david.villegas1@udea.edu.co>

**References**

Santos-Neto, M., Cysneiros, F. J. A., Leiva, V., & Ahmed, S. E. (2012). On new parameterizations of the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 28(1), 1-26.

**See Also**

[BS7](#).

**Examples**

```
# Example 1
# Plotting the mass function for different parameter values
curve(dBS7(x, mu=0.1, sigma=10),
      from=0.001, to=40,
      ylim=c(0, 0.20),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dBS7(x, mu=0.5, sigma=10),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=0.1, sigma=10",
                            "mu=0.5, sigma=10"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pBS7(x, mu=0.1, sigma=10),
      from=0.00001, to=50,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pBS7(x, mu=0.5, sigma=10),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=0.1, sigma=10",
                              "mu=0.5, sigma=10"),
      col=c("royalblue1", "tomato"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qBS7(p, mu=0.1, sigma=10), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pBS7(x, mu=0.1, sigma=10),
```

```

      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rBS7(n=10000, mu=0.1, sigma=10)
hist(x, freq=FALSE)
curve(dBS7(x, mu=0.1, sigma=10),
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hBS7(x, mu=0.1, sigma=10), from=0.001, to=60,
      col="tomato", ylab="Hazard function", las=1)

```

---

dCJ2

*The two-parameter Chris-Jerry distribution*


---

### Description

Density, distribution function, quantile function, random generation and hazard function for the two-parameter Chris-Jerry distribution with parameters  $\mu$  and  $\sigma$ .

### Usage

```

dCJ2(x, mu, sigma, log = FALSE)

pCJ2(q, mu, sigma, log.p = FALSE, lower.tail = TRUE)

qCJ2(p, mu, sigma, lower.tail = TRUE, log.p = FALSE)

rCJ2(n, mu, sigma)

hCJ2(x, mu, sigma, log = FALSE)

```

### Arguments

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
log, log.p	logical; if TRUE, probabilities p are given as $\log(p)$ .
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

## Details

The two-parameter Chris-Jerry distribution with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x; \sigma, \mu) = \frac{\mu^2}{\sigma\mu+2}(\sigma + \mu x^2)e^{-\mu x}; \quad x > 0, \quad \mu > 0, \quad \sigma > 0$$

Note: In this implementation we changed the original parameters  $\theta$  for  $\mu$  and  $\lambda$  for  $\sigma$ , we did it to implement this distribution within `gamlss` framework.

## Value

`dCJ2` gives the density, `pCJ2` gives the distribution function, `qCJ2` gives the quantile function, `rCJ2` generates random deviates and `hCJ2` gives the hazard function.

## Author(s)

Manuel Gutierrez Tangarife, <mgutierrezta@unal.edu.co>

## References

Chinedu, Eberechukwu Q., et al. "New lifetime distribution with applications to single acceptance sampling plan and scenarios of increasing hazard rates" *Symmetry* 15.10 (2023): 188.

## See Also

[CJ2](#)

## Examples

```
# Example 1
# Plotting the density function for different parameter values
curve(dCJ2(x, mu=3.5, sigma=0.01),
      from=0.0001, to=5,
      ylim=c(0, 1),
      col="red", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dCJ2(x, mu=2, sigma=0.05),
      col="green",
      lwd=2,
      add=TRUE)
curve(dCJ2(x, mu=1.5, sigma=0.01),
      col="blue",
      lwd=2,
      add=TRUE)
curve(dCJ2(x, mu=2.5, sigma=0.01),
      col="lightblue",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=3.5, sigma=0.01",
                            "mu=2, sigma=0.05",
                            "mu=1.5, sigma=0.01",
                            "mu=2.5, sigma=0.1"),
```

```

col=c( "red", "green","blue","lightblue"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pCJ2(x, mu=2.7, sigma=0.1),
      from=0.0001, to=5,
      ylim=c(0, 1),
      col="red", lwd=2,
      main="Cumulative function",
      xlab="x", ylab="f(x)")
curve(pCJ2(x, mu=2.3, sigma=0.5),
      col="green",
      lwd=2,
      add=TRUE)
curve(pCJ2(x, mu=2.8, sigma=0.2),
      col="blue",
      lwd=2,
      add=TRUE)
curve(pCJ2(x, mu=3.8, sigma=0.3),
      col="lightblue",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=2.75, sigma=0.1",
                              "mu=2.3, sigma=0.5",
                              "mu=2.8, sigma=0.2",
                              "mu=3.8, sigma=0.3"),
      col=c( "red", "green","blue","lightblue"), lwd=2, cex=0.6)

# Example 3
# Checking the quantile function
p <- seq(from=0.0001, to=0.99999, length.out=100)
plot(x=qCJ2(p, mu=2.3, sigma=1.7), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pCJ2(x, mu=2.3, sigma=1.7),
      from=0.0001, add=TRUE, col="red", lwd=2.5)

# Example 4
# Comparing the random generator output with
# the theoretical probabilities
x <- rCJ2(n=10000, mu=1.5, sigma=2.5)
hist(x, freq=FALSE)
curve(dCJ2(x, mu=1.5, sigma=2.5), from=0.001, to=8,
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hCJ2(x, mu=0.85, sigma=0.15),
      from=0.0001, to=5,
      ylim=c(0, 1),
      col="red", lwd=2,
      main="Hazard function",
      xlab="x", ylab="f(x)")
curve(hCJ2(x, mu=1, sigma=0.05),

```

```

      col="green",
      lwd=2,
      add=TRUE)
curve(hCJ2(x, mu=0.9, sigma=0.1),
      col="blue",
      lwd=2,
      add=TRUE)
curve(hCJ2(x, mu=1.15, sigma=0.1),
      col="lightblue",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=0.85, sigma=0.15",
                              "mu=1, sigma=0.05",
                              "mu=0.9, sigma=0.1",
                              "mu=1.15, sigma=0.1"),
      col=c("red", "green", "blue", "lightblue"), lwd=2, cex=0.5)

```

dCS2e

*The Cosine Sine Exponential distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Cosine Sine Exponential distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```

dCS2e(x, mu, sigma, nu, log = FALSE)

pCS2e(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

qCS2e(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

rCS2e(n, mu, sigma, nu)

hCS2e(x, mu, sigma, nu)

```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter.
<code>sigma</code>	parameter.
<code>nu</code>	parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

## Details

The Cosine Sine Exponential Distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(x) = \frac{\pi \sigma \mu \exp\left(\frac{-x}{\nu}\right)}{2\nu\left[\left(\mu \sin\left(\frac{\pi}{2} \exp\left(\frac{-x}{\nu}\right)\right) + \sigma \cos\left(\frac{\pi}{2} \exp\left(\frac{-x}{\nu}\right)\right)\right]^2},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

## Value

dCS2e gives the density, pCS2e gives the distribution function, qCS2e gives the quantile function, rCS2e generates random deviates and hCS2e gives the hazard function.

## Author(s)

Juan Pablo Ramirez

## References

Chesneau, C., Bakouch, H. S., & Hussain, T. (2019). A new class of probability distributions via cosine and sine functions with applications. *Communications in Statistics-Simulation and Computation*, 48(8), 2287-2300.

## See Also

[CS2e](#)

## Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
par(mfrow=c(1,1))
curve(dCS2e(x, mu=1, sigma=0.1, nu =0.1), from=0, to=1,
      ylim=c(0, 3), col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pCS2e(x, mu=1, sigma=0.1, nu =0.1),
      from=0, to=1, col="red", las=1, ylab="F(x)")
curve(pCS2e(x, mu=1, sigma=0.1, nu =0.1, lower.tail=FALSE),
      from=0, to=1, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qCS2e(p, mu=0.1, sigma=1, nu=0.1), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pCS2e(x, mu=0.1, sigma=1, nu=0.1), from=0, add=TRUE, col="red")

## The random function
hist(rCS2e(n=10000, mu=0.1, sigma=1, nu=0.1), freq=FALSE,
     xlab="x", las=1, main="")
curve(dCS2e(x, mu=0.1, sigma=1, nu=0.1), from=0, add=TRUE, col="red")
```

```
## The Hazard function
par(mfrow=c(1,1))
curve(hCS2e(x, mu=1, sigma=0.1, nu =0.1), from=0, to=1, ylim=c(0, 10),
      col=2, ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters
```

dEEG

*The Extended Exponential Geometric distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Extended Exponential Geometric distribution with parameters  $\mu$  and  $\sigma$ .

**Usage**

```
dEEG(x, mu, sigma, log = FALSE)
pEEG(q, mu, sigma, lower.tail = TRUE, log.p = FALSE)
qEEG(p, mu, sigma, lower.tail = TRUE, log.p = FALSE)
rEEG(n, mu, sigma)
hEEG(x, mu, sigma)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter.
<code>sigma</code>	parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

**Details**

The Extended Exponential Geometric distribution with parameters  $\mu$ , and  $\sigma$  has density given by

$$f(x) = \mu\sigma \exp(-\mu x)(1 - (1 - \sigma) \exp(-\mu x))^{-2},$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ .

**Value**

dEEG gives the density, pEEG gives the distribution function, qEEG gives the quantile function, rEEG generates random deviates and hEEG gives the hazard function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. Reliability Engineering & System Safety, 124, 32-55.

Adamidis, K., Dimitrakopoulou, T., & Loukas, S. (2005). On an extension of the exponential-geometric distribution. Statistics & probability letters, 73(3), 259-269.

**See Also**

[EEG](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
par(mfrow=c(1,1))
curve(dEEG(x, mu = 1, sigma =3), from = 0, to = 10,
      col = "red", las = 1, ylab = "f(x)")

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pEEG(x, mu = 1, sigma =3), from = 0, to = 10,
      ylim = c(0, 1), col = "red", las = 1, ylab = "F(x)")
curve(pEEG(x, mu = 1, sigma =3, lower.tail = FALSE),
      from = 0, to = 6, ylim = c(0, 1), col = "red", las = 1, ylab = "R(x)")

## The quantile function
p <- seq(from = 0, to = 0.99999, length.out = 100)
plot(x = qEEG(p = p, mu = 1, sigma =0.5), y = p,
     xlab = "Quantile", las = 1, ylab = "Probability")
curve(pEEG(x, mu = 1, sigma =0.5), from = 0, add = TRUE,
     col = "red")

## The random function
hist(rEEG(1000, mu = 1, sigma =1), freq = FALSE, xlab = "x",
     ylim = c(0, 0.9), las = 1, main = "")
curve(dEEG(x, mu = 1, sigma =1), from = 0, add = TRUE,
     col = "red", ylim = c(0, 0.8))

## The Hazard function
par(mfrow=c(1,1))
curve(hEEG(x, mu = 1, sigma =0.5), from = 0, to = 2,
```

```
col = "red", ylab = "Hazard function", las = 1)
par(old_par) # restore previous graphical parameters
```

dEGG

*The four parameter Exponentiated Generalized Gamma distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the four parameter Exponentiated Generalized Gamma distribution with parameters mu, sigma, nu and tau.

**Usage**

```
dEGG(x, mu, sigma, nu, tau, log = FALSE)
pEGG(q, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)
qEGG(p, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)
rEGG(n, mu, sigma, nu, tau)
hEGG(x, mu, sigma, nu, tau)
```

**Arguments**

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
nu	parameter.
tau	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

Four-Parameter Exponentiated Generalized Gamma distribution with parameters mu, sigma, nu and tau has density given by

$$f(x) = \frac{\nu\sigma}{\mu\Gamma(\tau)} \left(\frac{x}{\mu}\right)^{\sigma\tau-1} \exp\left\{-\left(\frac{x}{\mu}\right)^\sigma\right\} \left\{\gamma_1\left(\tau, \left(\frac{x}{\mu}\right)^\sigma\right)\right\}^{\nu-1},$$

for  $x > 0$ .

**Value**

dEGG gives the density, pEGG gives the distribution function, qEGG gives the quantile function, rEGG generates random deviates and hEGG gives the hazard function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Cordeiro, G. M., Ortega, E. M., & Silva, G. O. (2011). The exponentiated generalized gamma distribution with application to lifetime data. *Journal of statistical computation and simulation*, 81(7), 827-842.

**See Also**

[EGG](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dEGG(x, mu=0.1, sigma=0.8, nu=10, tau=1.5), from=0.000001, to=1.5, ylim=c(0, 2.5),
      col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pEGG(x, mu=0.1, sigma=0.8, nu=10, tau=1.5),
      from=0.000001, to=1.5, col="red", las=1, ylab="F(x)")
curve(pEGG(x, mu=0.1, sigma=0.8, nu=10, tau=1.5, lower.tail=FALSE),
      from=0.000001, to=1.5, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qEGG(p, mu=0.1, sigma=0.8, nu=10, tau=1.5), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pEGG(x, mu=0.1, sigma=0.8, nu=10, tau=1.5),
      from=0.00001, add=TRUE, col="red")

## The random function
hist(rEGG(n=100, mu=0.1, sigma=0.8, nu=10, tau=1.5), freq=FALSE,
     xlab="x", las=1, main="")
curve(dEGG(x, mu=0.1, sigma=0.8, nu=10, tau=1.5),
      from=0.0001, to=2, add=TRUE, col="red")

## The Hazard function
curve(hEGG(x, mu=0.1, sigma=0.8, nu=10, tau=1.5), from=0.0001, to=1.5,
     col="red", ylab="Hazard function", las=1)
```

```
par(old_par) # restore previous graphical parameters
```

---

dEMWEx

*The Exponentiated Modified Weibull Extension distribution*


---

## Description

Density, distribution function, quantile function, random generation and hazard function for the Exponentiated Modified Weibull Extension distribution with parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$ .

## Usage

```
dEMWEx(x, mu, sigma, nu, tau, log = FALSE)
pEMWEx(q, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)
qEMWEx(p, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)
rEMWEx(n, mu, sigma, nu, tau)
hEMWEx(x, mu, sigma, nu, tau)
```

## Arguments

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
nu	parameter.
tau	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

## Details

The Exponentiated Modified Weibull Extension Distribution with parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$  has density given by

$$f(x) = \nu\sigma\tau\left(\frac{x}{\mu}\right)^{\sigma-1} \exp\left(\left(\frac{x}{\mu}\right)^\sigma + \nu\mu(1 - \exp\left(\left(\frac{x}{\mu}\right)^\sigma\right))\right)(1 - \exp(\nu\mu(1 - \exp\left(\left(\frac{x}{\mu}\right)^\sigma\right))))^{\tau-1},$$

for  $x > 0$ ,  $\nu > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\tau > 0$ .

## Value

dEMWEx gives the density, pEMWEx gives the distribution function, qEMWEx gives the quantile function, rEMWEx generates random deviates and hEMWEx gives the hazard function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Sarhan, A. M., & Apaloo, J. (2013). Exponentiated modified Weibull extension distribution. *Reliability Engineering & System Safety*, 112, 137-144.

**See Also**

[EMWEx](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dEMWEx(x, mu = 49.046, sigma =3.148, nu=0.00005, tau=0.1), from=0, to=100,
      col = "red", las = 1, ylab = "f(x)")

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pEMWEx(x, mu = (1/4), sigma =1, nu=1, tau=2), from = 0, to = 1,
      ylim = c(0, 1), col = "red", las = 1, ylab = "F(x)")
curve(pEMWEx(x, mu = (1/4), sigma =1, nu=1, tau=2, lower.tail = FALSE),
      from = 0, to = 1, ylim = c(0, 1), col = "red", las = 1, ylab = "R(x)")

## The quantile function
p <- seq(from = 0, to = 0.99999, length.out = 100)
plot(x = qEMWEx(p = p, mu = 49.046, sigma =3.148, nu=0.00005, tau=0.1), y = p,
     xlab = "Quantile", las = 1, ylab = "Probability")
curve(pEMWEx(x, mu = 49.046, sigma =3.148, nu=0.00005, tau=0.1), from = 0, add = TRUE,
     col = "red")

## The random function
hist(rEMWEx(1000, mu = (1/4), sigma =1, nu=1, tau=2), freq = FALSE, xlab = "x",
     las = 1, main = "")
curve(dEMWEx(x, mu = (1/4), sigma =1, nu=1, tau=2), from = 0, add = TRUE,
     col = "red", ylim = c(0, 0.5))

## The Hazard function(
par(mfrow=c(1,1))
curve(hEMWEx(x, mu = 49.046, sigma =3.148, nu=0.00005, tau=0.1), from = 0, to = 80,
     col = "red", ylab = "Hazard function", las = 1)

par(old_par) # restore previous graphical parameters
```

dEOFNH

*The Extended Odd Frechet-Nadarajah-Haghighi***Description**

Density, distribution function, quantile function, random generation and hazard function for the Extended Odd Fr?chet-Nadarajah-Haghighi distribution with parameters mu, sigma, nu and tau.

**Usage**

```
dEOFNH(x, mu, sigma, nu, tau, log = FALSE)
```

```
pEOFNH(q, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)
```

```
qEOFNH(p, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)
```

```
rEOFNH(n, mu, sigma, nu, tau)
```

```
hEOFNH(x, mu, sigma, nu, tau)
```

**Arguments**

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
nu	parameter.
tau	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

The Extended Odd Frechet-Nadarajah-Haghighi mu, sigma, nu and tau has density given by

$$f(x) = \frac{\mu\sigma\nu\tau(1+\nu x)^{\sigma-1}e^{(1-(1+\nu x)^\sigma)}[1-(1-e^{(1-(1+\nu x)^\sigma)})^\mu]^{\tau-1}}{(1-e^{(1-(1+\nu x)^\sigma)})^{\mu\tau+1}}e^{-[(1-e^{(1-(1+\nu x)^\sigma)})^{-\mu}-1]^\tau},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$ ,  $\nu > 0$  and  $\tau > 0$ .

**Value**

dEOFNH gives the density, pEOFNH gives the distribution function, qEOFNH gives the quantile function, rEOFNH generates random numbers and hEOFNH gives the hazard function.

**Author(s)**

Helber Santiago Padilla, <hspadillar@unal.edu.co>

**References**

Nasiru, S. (2018). Extended Odd Fréchet-G Family of Distributions Journal of Probability and Statistics, 2018(1), 2931326.

**See Also**

[EOFNH](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

##The probability density function
par(mfrow=c(1, 1))

curve(dEOFNH(x, mu=18.5, sigma=5.1, nu=0.1, tau=0.1),
      from=0, to=10, ylim=c(0, 0.25),
      col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pEOFNH(x,mu=18.5, sigma=5.1, nu=0.1, tau=0.1), from = 0, to = 10,
      ylim = c(0, 1), col = "red", las = 1, ylab = "F(x)")
curve(pEOFNH(x, mu=18.5, sigma=5.1, nu=0.1, tau=0.1, lower.tail = FALSE),
      from = 0, to = 10, ylim = c(0, 1), col = "red", las = 1, ylab = "R(x)")

##The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qEOFNH(p, mu=18.5, sigma=5.1, nu=0.1, tau=0.1), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pEOFNH(x, mu=18.5, sigma=5.1, nu=0.1, tau=0.1), from=0, add=TRUE, col="red")

##The random function
hist(rEOFNH(n=10000, mu=18.5, sigma=5.1, nu=0.1, tau=0.1), freq=FALSE,
     xlab="x", las=1, main="")
curve(dEOFNH(x, mu=18.5, sigma=5.1, nu=0.1, tau=0.1), from=0, add=TRUE, col="red", ylim=c(0,1.25))

##The Hazard function
par(mfrow=c(1,1))
curve(hEOFNH(x, mu=18.5, sigma=5.1, nu=0.1, tau=0.1), from=0, to=10, ylim=c(0, 1),
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters
```

dEW

*The Exponentiated Weibull distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the exponentiated Weibull distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```
dEW(x, mu, sigma, nu, log = FALSE)
```

```
pEW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
```

```
qEW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
```

```
rEW(n, mu, sigma, nu)
```

```
hEW(x, mu, sigma, nu)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>mu</code>	scale parameter.
<code>sigma, nu</code>	shape parameters.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

**Details**

The Exponentiated Weibull Distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(x) = \nu\mu\sigma x^{\sigma-1} \exp(-\mu x^\sigma)(1 - \exp(-\mu x^\sigma))^{\nu-1},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

**Value**

dEW gives the density, pEW gives the distribution function, qEW gives the quantile function, rEW generates random deviates and hEW gives the hazard function.

**See Also**

[EW](#)

**Examples**

```

old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dEW(x, mu=2, sigma=1.5, nu=0.5), from=0, to=2,
      ylim=c(0, 2.5), col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pEW(x, mu=2, sigma=1.5, nu=0.5),
      from=0, to=2, col="red", las=1, ylab="F(x)")
curve(pEW(x, mu=2, sigma=1.5, nu=0.5, lower.tail=FALSE),
      from=0, to=2, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qEW(p, mu=2, sigma=1.5, nu=0.5), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pEW(x, mu=2, sigma=1.5, nu=0.5), from=0, add=TRUE, col="red")

## The random function
hist(rEW(n=10000, mu=2, sigma=1.5, nu=0.5), freq=FALSE,
     xlab="x", las=1, main="")
curve(dEW(x, mu=2, sigma=1.5, nu=0.5), from=0, add=TRUE, col="red")

## The Hazard function
par(mfrow=c(1,1))
curve(hEW(x, mu=2, sigma=1.5, nu=0.5), from=0, to=2, ylim=c(0, 7),
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters

```

---

dEXL

*The exponentiated XLindley distribution*


---

**Description**

Density, distribution function, quantile function, random generation and hazard function for the exponentiated XLindley distribution with parameters  $\mu$  and  $\sigma$ .

**Usage**

```

dEXL(x, mu, sigma, log = FALSE)

pEXL(q, mu, sigma, log.p = FALSE, lower.tail = TRUE)

qEXL(p, mu, sigma, lower.tail = TRUE, log.p = FALSE)

rEXL(n, mu, sigma)

```

```
hEXL(x, mu, sigma, log = FALSE)
```

### Arguments

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

### Details

The exponentiated XLindley with parameters mu and sigma has density given by

$$f(x) = \frac{\sigma\mu^2(2+\mu+x)\exp(-\mu x)}{(1+\mu)^2} \left[ 1 - \left( 1 + \frac{\mu x}{(1+\mu)^2} \right) \exp(-\mu x) \right]^{\sigma-1}$$

for  $x \geq 0$ ,  $\mu \geq 0$  and  $\sigma \geq 0$ .

Note: In this implementation we changed the original parameters  $\delta$  for  $\mu$  and  $\alpha$  for  $\sigma$ , we did it to implement this distribution within gamlss framework.

### Value

dEXL gives the density, pEXL gives the distribution function, qEXL gives the quantile function, rEXL generates random deviates and hEXL gives the hazard function.

### Author(s)

Manuel Gutierrez Tangarife, <mgutierrez@unal.edu.co>

### References

Alomair, A. M., Ahmed, M., Tariq, S., Ahsan-ul-Haq, M., & Talib, J. (2024). An exponentiated XLindley distribution with properties, inference and applications. *Heliyon*, 10(3).

### See Also

[EXL](#).

### Examples

```
# Example 1
# Plotting the mass function for different parameter values
curve(dEXL(x, mu=0.5, sigma=0.5),
      from=0, to=5,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
```

```

    main="Density function",
    xlab="x", ylab="f(x)")
curve(dEXL(x, mu=1, sigma=0.5),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(dEXL(x, mu=1.5, sigma=0.5),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=0.5, sigma=0.5",
                            "mu=1.0, sigma=0.5",
                            "mu=1.5, sigma=0.5"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.6)

curve(dEXL(x, mu=0.5, sigma=1),
      from=0, to=5,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dEXL(x, mu=1, sigma=1),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(dEXL(x, mu=1.5, sigma=1),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=0.5, sigma=1",
                            "mu=1.0, sigma=1",
                            "mu=1.5, sigma=1"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.6)

curve(dEXL(x, mu=0.5, sigma=1.5),
      from=0., to=8,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dEXL(x, mu=1, sigma=1.5),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(dEXL(x, mu=1.5, sigma=1.5),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=0.5, sigma=1.5",
                            "mu=1.0, sigma=1.5",
                            "mu=1.5, sigma=1.5"),

```

```

        col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1
curve(pEXL(x, mu=0.5, sigma=0.5),
      from=0, to=5,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="f(x)")
curve(pEXL(x, mu=1, sigma=0.5),
      col="tomato",
      lwd=2,
      add=TRUE)
curve(pEXL(x, mu=1.5, sigma=0.5),
      col="seagreen",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=0.5, sigma=0.5",
                                "mu=1.0, sigma=0.5",
                                "mu=1.5, sigma=0.5"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qEXL(p, mu=2.3, sigma=1.7), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pEXL(x, mu=2.3, sigma=1.7),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Comparing quantile, density and cumulative
p <- c(0.25, 0.5, 0.75)
quantile <- qEXL(p=p, mu=2.3, sigma=1.7)

for(i in quantile){
  print(integrate(dEXL, lower=0, upper=i, mu=2.3, sigma=1.7))
}

pEXL(q=quantile, mu=2.3, sigma=1.7)

# Example 4
# The random function
x <- rEXL(n=10000, mu=1.5, sigma=2.5)
hist(x, freq=FALSE)
curve(dEXL(x, mu=1.5, sigma=2.5), from=0, to=20,
      add=TRUE, col="tomato", lwd=2)

# Example 5
# The Hazard function
curve(hEXL(x, mu=1.5, sigma=2), from=0.001, to=4,
      col="tomato", ylab="Hazard function", las=1)

```

dExW

*The Extended Weibull distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Extended Weibull distribution with parameters mu, sigma and nu.

**Usage**

```
dExW(x, mu, sigma, nu, log = FALSE)
```

```
pExW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
```

```
qExW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
```

```
rExW(n, mu, sigma, nu)
```

```
hExW(x, mu, sigma, nu)
```

**Arguments**

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
nu	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

The Extended Weibull distribution with parameters mu, sigma and nu has density given by

$$f(x) = \frac{\mu\sigma\nu x^{\sigma-1} \exp(-\mu x^\sigma)}{[1 - (1-\nu)\exp(-\mu x^\sigma)]^2},$$

for  $x > 0$ .

**Value**

dExW gives the density, pExW gives the distribution function, qExW gives the quantile function, rExW generates random deviates and hExW gives the hazard function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

## References

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Zhang, T., & Xie, M. (2007). Failure data analysis with extended Weibull distribution. *Communications in Statistics—Simulation and Computation*, 36(3), 579-592.

## See Also

[ExW](#)

## Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dExW(x, mu=0.3, sigma=2, nu=0.05), from=0.0001, to=2,
      col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pExW(x, mu=0.3, sigma=2, nu=0.05),
      from=0.0001, to=2, col="red", las=1, ylab="F(x)")
curve(pExW(x, mu=0.3, sigma=2, nu=0.05, lower.tail=FALSE),
      from=0.0001, to=2, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qExW(p, mu=0.3, sigma=2, nu=0.05), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pExW(x, mu=0.3, sigma=2, nu=0.05),
     from=0, add=TRUE, col="red")

## The random function
hist(rExW(n=10000, mu=0.3, sigma=2, nu=0.05), freq=FALSE,
     xlab="x", ylim=c(0, 2), las=1, main="")
curve(dExW(x, mu=0.3, sigma=2, nu=0.05),
     from=0.001, to=4, add=TRUE, col="red")

## The Hazard function
curve(hExW(x, mu=0.3, sigma=2, nu=0.05), from=0.001, to=4,
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters
```

**Description**

These functions define the density, distribution function, quantile function and random generation for the Ex-Wald distribution with parameter  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```
dExWALD(x, mu = 1.5, sigma = 1.5, nu = 2, log = FALSE)
```

```
pExWALD(q, mu = 1.5, sigma = 1.5, nu = 2, lower.tail = TRUE, log.p = FALSE)
```

```
qExWALD(p, mu = 1.5, sigma = 1.5, nu = 2)
```

```
rExWALD(n, mu = 1.5, sigma = 1.5, nu = 2)
```

**Arguments**

x, q	vector of (non-negative integer) quantiles.
mu	vector of the mu parameter.
sigma	vector of the sigma parameter.
nu	vector of the nu parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of random values to return.

**Details**

The Wald distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(x|\mu, \sigma, \nu) = \frac{1}{\nu} \exp\left(\frac{-x}{\nu} + \sigma(\mu - k)\right) F_W(x|k, \sigma) \text{ for } k \geq 0$$

$$f(x|\mu, \sigma, \nu) = \frac{1}{\nu} \exp\left(\frac{-(\sigma-\mu)^2}{2x}\right) Re\left(w(k' \sqrt{x/2} + \frac{\sigma i}{\sqrt{2x}})\right) \text{ for } k < 0$$

where  $k = \sqrt{\mu^2 - \frac{2}{\nu}}$ ,  $k' = \sqrt{\frac{2}{\nu} - \mu^2}$  and  $F_W$  corresponds to the cumulative function of the Wald distribution.

More details about those expressions can be found on page 680 from Heathcote (2004).

**Value**

dExWALD gives the density, pExWALD gives the distribution function, qExWALD gives the quantile function, rExWALD generates random deviates.

**Author(s)**

Freddy Hernandez, <fhernanb@unal.edu.co>

## References

Schwarz, W. (2001). The ex-Wald distribution as a descriptive model of response times. *Behavior Research Methods, Instruments, & Computers*, 33, 457-469.

Heathcote, A. (2004). Fitting Wald and ex-Wald distributions to response time data: An example using functions for the S-PLUS package. *Behavior Research Methods, Instruments, & Computers*, 36, 678-694.

## See Also

[ExWALD](#)

## Examples

```
# Example 1
# Plotting the mass function for different parameter values
curve(dExWALD(x, mu=0.15, sigma=52.5, nu=50), ylim=c(0, 0.005),
      from=0, to=1200, col="cadetblue3", las=1, ylab="f(x)")

curve(dExWALD(x, mu=0.20, sigma=70, nu=50),
      add=TRUE, col= "purple")

curve(dExWALD(x, mu=0.25, sigma=87.5, nu=50),
      add=TRUE, col="goldenrod")

curve(dExWALD(x, mu=0.20, sigma=70, nu=115),
      add=TRUE, col="tomato")

curve(dExWALD(x, mu=0.20, sigma=70, nu=35),
      add=TRUE, col="blue")

legend("topright", col=c("cadetblue3", "purple", "goldenrod",
                        "tomato", "blue"),
      lty=1, bty="n",
      legend=c("mu=0.15, sigma=52.5, nu=50",
              "mu=0.20, sigma=70.0, nu=50",
              "mu=0.25, sigma=87.5, nu=50",
              "mu=0.20, sigma=70.0, nu=115",
              "mu=0.20, sigma=70.0, nu=35"))

# Example 2
# Checking if the cumulative curves converge to 1
curve(pExWALD(x, mu=0.15, sigma=52.5, nu=50), ylim=c(0, 1),
      from=0, to=1200, col="cadetblue3", las=1, ylab="F(x)")

curve(pExWALD(x, mu=0.20, sigma=70, nu=50),
      add=TRUE, col= "purple")

curve(pExWALD(x, mu=0.25, sigma=87.5, nu=50),
      add=TRUE, col="goldenrod")

curve(pExWALD(x, mu=0.20, sigma=70, nu=115),
```

```

    add=TRUE, col="tomato")

curve(pExWALD(x, mu=0.20, sigma=70, nu=35),
      add=TRUE, col="blue")

legend("bottomright", col=c("cadetblue3", "purple", "goldenrod",
                           "tomato", "blue"),
      lty=1, bty="n",
      legend=c("mu=0.15, sigma=52.5, nu=50",
              "mu=0.20, sigma=70.0, nu=50",
              "mu=0.25, sigma=87.5, nu=50",
              "mu=0.20, sigma=70.0, nu=115",
              "mu=0.20, sigma=70.0, nu=35"))

# Example 3
# Checking the quantile function
mu <- 5
sigma <- 3
nu <- 2
p <- seq(from=0.1, to=0.99, length.out=100)
plot(x=qExWALD(p, mu=mu, sigma=sigma, nu=nu), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pExWALD(x, mu=mu, sigma=sigma, nu=nu), from=0, add=TRUE, col="red")

# Example 4
# Comparing the random generator output with
# the theoretical probabilities
mu <- 0.2
sigma <- 70
nu <- 35
x <- rExWALD(n=10000, mu=mu, sigma=sigma, nu=nu)
hist(x, freq=FALSE)
curve(dExWALD(x, mu=mu, sigma=sigma, nu=nu), col="tomato", add=TRUE)

```

**Description**

Density, distribution function, quantile function, random generation and hazard function for the Flexible Weibull Extension distribution with parameters  $\mu$  and  $\sigma$ .

**Usage**

```
dFWE(x, mu, sigma, log = FALSE)
```

```
pFWE(q, mu, sigma, lower.tail = TRUE, log.p = FALSE)
```

```
qFWE(p, mu, sigma, lower.tail = TRUE, log.p = FALSE)
```

```
rFWE(n, mu, sigma)
```

```
hFWE(x, mu, sigma)
```

### Arguments

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

### Details

The Flexible Weibull extension with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x) = (\mu + \sigma/x^2) \exp(\mu x - \sigma/x) \exp(-\exp(\mu x - \sigma/x))$$

for  $x > 0$ .

### Value

dFWE gives the density, pFWE gives the distribution function, qFWE gives the quantile function, rFWE generates random deviates and hFWE gives the hazard function.

### See Also

[FWE](#)

### Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dFWE(x, mu=0.75, sigma=0.5), from=0, to=3,
      ylim=c(0, 1.7), col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pFWE(x, mu=0.75, sigma=0.5), from=0, to=3,
      col="red", las=1, ylab="F(x)")
curve(pFWE(x, mu=0.75, sigma=0.5, lower.tail=FALSE),
      from=0, to=3, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qFWE(p, mu=0.75, sigma=0.5), y=p, xlab="Quantile",
```

```

    las=1, ylab="Probability")
curve(pFWE(x, mu=0.75, sigma=0.5), from=0, add=TRUE, col="red")

## The random function
hist(rFWE(n=1000, mu=2, sigma=0.5), freq=FALSE, xlab="x",
     ylim=c(0, 2), las=1, main="")
curve(dFWE(x, mu=2, sigma=0.5), from=0, to=3, add=TRUE, col="red")

## The Hazard function
par(mfrow=c(1,1))
curve(hFWE(x, mu=0.75, sigma=0.5), from=0, to=2, ylim=c(0, 2.5),
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters

```

---

dGammaW

*The Gamma Weibull distribution*


---

## Description

Density, distribution function, quantile function, random generation and hazard function for the Gamma Weibull distribution with parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$ .

## Usage

```

dGammaW(x, mu, sigma, nu, log = FALSE)

pGammaW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

qGammaW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

rGammaW(n, mu, sigma, nu)

hGammaW(x, mu, sigma, nu)

```

## Arguments

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter.
<code>sigma</code>	parameter.
<code>nu</code>	parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

**Details**

The Gamma Weibull Distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(x) = \frac{\sigma \mu^\nu}{\Gamma(\nu)} x^{\nu\sigma-1} \exp(-\mu x^\sigma),$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

**Value**

dGammaW gives the density, pGammaW gives the distribution function, qGammaW gives the quantile function, rGammaW generates random deviates and hGammaW gives the hazard function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. Reliability Engineering & System Safety, 124, 32-55.

Stacy, E. W. (1962). A generalization of the gamma distribution. The Annals of mathematical statistics, 1187-1192.

**See Also**

[GammaW](#)

**Examples**

```
# Example 1
# Plotting the mass function for different parameter values

## The probability density function
curve(dGammaW(x, mu=2, sigma=1.5, nu=0.5),
      from=0, to=2,
      col="red", lwd=2,
      main="Density function",
      xlab="x", ylab="f(x)")
curve(dGammaW(x, mu=2.4, sigma=1.5, nu=1.3),
      col="blue",
      lwd=2,
      add=TRUE)
legend("topright", legend=c("mu=2.0, sigma=1.5, nu=0.5",
                           "mu=2.4, sigma=1.5, nu=1.3"),
      col=c("red", "blue"), lwd=2, cex=0.6)

# Example 2
# Checking if the cumulative curves converge to 1

curve(pGammaW(x, mu=0.5, sigma=2, nu=1),
      from=0, to=3,
      col="red", lwd=2, ylab="F(x)")
```

```

curve(pGammaW(x, mu=2.4, sigma=1.5, nu=1.3),
      col="blue",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=2.0, sigma=1.5, nu=0.5",
                               "mu=2.4, sigma=1.5, nu=1.3"),
      col=c("red", "blue"), lwd=2, cex=0.6)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qGammaW(p, mu=2.3, sigma=1.7, nu=1.2), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pGammaW(x, mu=2.3, sigma=1.7, nu=1.2),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
x <- rGammaW(n=10000, mu=2.4, sigma=1.5, nu=1.3)
hist(x, freq=FALSE)
curve(dGammaW(x, mu=2.4, sigma=1.5, nu=1.3),
      add=TRUE, col="tomato", lwd=2)

# The Hazard function
curve(hGammaW(x, mu=2.4, sigma=1.5, nu=1.3), from=0, to=5,
      col="red", ylab="Hazard function", las=1)

```

---

dGGD

*The Generalized Gompertz distribution*


---

## Description

Density, distribution function, quantile function, random generation and hazard function for the generalized Gompertz distribution with parameters  $\mu$   $\sigma$  and  $\nu$ .

## Usage

```

dGGD(x, mu, sigma, nu, log = FALSE)

pGGD(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

qGGD(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

rGGD(n, mu, sigma, nu)

hGGD(x, mu, sigma, nu)

```

**Arguments**

x, q	vector of quantiles.
mu, nu	scale parameter.
sigma	shape parameters.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

The Generalized Gompertz Distribution with parameters mu, sigma and nu has density given by

$$f(x) = \nu \mu \exp\left(-\frac{\mu}{\sigma}(\exp(\sigma x - 1))\right) \left(1 - \exp\left(-\frac{\mu}{\sigma}(\exp(\sigma x - 1))\right)\right)^{(\nu-1)},$$

for  $x \geq 0$ ,  $\mu > 0$ ,  $\sigma \geq 0$  and  $\nu > 0$ .

**Value**

dGGD gives the density, pGGD gives the distribution function, qGGD gives the quantile function, rGGD generates random deviates and hGGD gives the hazard function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

El-Gohary, A., Alshamrani, A., & Al-Otaibi, A. N. (2013). The generalized Gompertz distribution. Applied mathematical modelling, 37(1-2), 13-24.

**See Also**

[GGD](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
par(mfrow = c(1, 1))
curve(dGGD(x, mu=1, sigma=0.3, nu=1.5), from = 0, to = 4,
      col = "red", las = 1, ylab = "f(x)")

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pGGD(x, mu=1, sigma=0.3, nu=1.5), from = 0, to = 4,
      ylim = c(0, 1), col = "red", las = 1, ylab = "F(x)")
curve(pGGD(x, mu=1, sigma=0.3, nu=1.5, lower.tail = FALSE),
      from = 0, to = 4, ylim = c(0, 1), col = "red", las = 1, ylab = "R(x)")
```

```

## The quantile function
p <- seq(from = 0, to = 0.99999, length.out = 100)
plot(x = qGGD(p=p, mu=1, sigma=0.3, nu=1.5), y = p,
     xlab = "Quantile", las = 1, ylab = "Probability")
curve(pGGD(x, mu=1, sigma=0.3, nu=1.5), from = 0, add = TRUE,
      col = "red")

## The random function
hist(rGGD(1000, mu=1, sigma=0.3, nu=1.5), freq = FALSE, xlab = "x",
     las = 1, ylim = c(0, 0.7), main = "")
curve(dGGD(x,mu=1, sigma=0.3, nu=1.5), from = 0, to =8, add = TRUE,
      col = "red")

## The Hazard function
par(mfrow=c(1,1))
curve(hGGD(x, mu=1, sigma=0.3, nu=1.5), from = 0, to = 3, col = "red",
     ylab = "The hazard function", las = 1)

par(old_par) # restore previous graphical parameters

```

---

dGIW

*The Generalized Inverse Weibull distribution*


---

## Description

Density, distribution function, quantile function, random generation and hazard function for the Generalized Inverse Weibull distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

## Usage

```

dGIW(x, mu, sigma, nu, log = FALSE)

pGIW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

qGIW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

rGIW(n, mu, sigma, nu)

hGIW(x, mu, sigma, nu)

```

## Arguments

$x, q$	vector of quantiles.
$\mu$	parameter.
$\sigma$	parameter.
$\nu$	parameter.
$\log, \log.p$	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .

lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

### Details

The Generalized Inverse Weibull distribution mu, sigma and nu has density given by

$$f(x) = \nu\sigma\mu^\sigma x^{-(\sigma+1)} \exp\left\{-\nu\left(\frac{\mu}{x}\right)^\sigma\right\},$$

for  $x > 0$ .

### Value

dGIW gives the density, pGIW gives the distribution function, qGIW gives the quantile function, rGIW generates random deviates and hGIW gives the hazard function.

### Author(s)

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

### References

- Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. Reliability Engineering & System Safety, 124, 32-55.
- De Gusmao, F. R., Ortega, E. M., & Cordeiro, G. M. (2011). The generalized inverse Weibull distribution. Statistical Papers, 52, 591-619.

### See Also

[GIW](#)

### Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dGIW(x, mu=3, sigma=5, nu=0.5), from=0.001, to=8,
      col="red", ylab="f(x)", las=1)

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pGIW(x, mu=3, sigma=5, nu=0.5),
      from=0.0001, to=14, col="red", las=1, ylab="F(x)")
curve(pGIW(x, mu=3, sigma=5, nu=0.5, lower.tail=FALSE),
      from=0.0001, to=14, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qGIW(p, mu=3, sigma=5, nu=0.5), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pGIW(x, mu=3, sigma=5, nu=0.5),
```

```

    from=0, add=TRUE, col="red")

## The random function
hist(rGIW(n=1000, mu=3, sigma=5, nu=0.5), freq=FALSE,
     xlab="x", ylim=c(0, 0.8), las=1, main="")
curve(dGIW(x, mu=3, sigma=5, nu=0.5),
      from=0.001, to=14, add=TRUE, col="red")

## The Hazard function
curve(hGIW(x, mu=3, sigma=5, nu=0.5), from=0.001, to=30,
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters

```

---

dGL2

*The Generalized Lindley Type II (GL2) distribution*


---

### Description

These functions define the density, distribution function, quantile function and random generation for the Generalized Lindley Type II, GL2(), distribution with parameters  $\mu$  and  $\sigma$ .

### Usage

```

dGL2(x, mu, sigma, log = FALSE)

pGL2(q, mu, sigma, lower.tail = TRUE, log.p = FALSE)

qGL2(p, mu, sigma, lower.tail = TRUE, log.p = FALSE)

rGL2(n, mu, sigma)

hGL2(x, mu = 0.5, sigma = 0.5)

```

### Arguments

x, q	vector of positive quantiles.
mu	vector of the mu parameter.
sigma	vector of the sigma parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of random values to return.

## Details

The Generalized Lindley Type II distribution with parameters  $\mu > 0$  and  $\sigma > 0$  has support  $x > 0$  and probability density function given by

$$f(x|\mu, \sigma) = \frac{\mu^2}{\mu+1} \left( 1 + \frac{\mu^{\sigma-2} x^{\sigma-1}}{\Gamma(\sigma)} \right) e^{-\mu x},$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ .

Note: in this implementation we changed the original parameters  $\theta$  for  $\mu$  and  $\alpha$  for  $\sigma$ . This reparameterization was performed to implement the distribution within the GAMLSS framework.

The GL2 distribution is a flexible two-parameter extension of the classical Lindley distribution and is suitable for modeling positive lifetime and survival data.

## Value

dGL2 gives the density, pGL2 gives the distribution function, qGL2 gives the quantile function, and rGL2 generates random deviates.

## Author(s)

Sofia Cadavid Rueda, <socadavidr@unal.edu.co>

## References

Ekhosuehi, N., Opone, F., & Odobaire, F. (2018). A New Generalized Two Parameter Lindley Distribution. *Journal of the Nigerian Statistical Association*, 30, 547–566.

## See Also

[GL2](#).

## Examples

```
# Example 1
# Plotting the mass function for different parameter values
x_vals <- seq(0, 6, length.out = 500)
# Calculate densities
d1 <- dGL2(x_vals, mu = 0.7, sigma = 1.4)
d2 <- dGL2(x_vals, mu = 0.3, sigma = 1.2)
d3 <- dGL2(x_vals, mu = 3.0, sigma = 1.0)
d4 <- dGL2(x_vals, mu = 6.0, sigma = 1.0)
# Plot
plot(x_vals, d1, type = "l", col = "red", lwd = 2, lty = 1,
      ylim = c(0, 5), xlim = c(0, 6),
      xlab = "x", ylab = "f(x)",
      main = "Probability Density Function of the GL2 Distribution",
      las = 1)
lines(x_vals, d2, col = "black", lwd = 2, lty = 2)
lines(x_vals, d3, col = "yellow", lwd = 2, lty = 1)
lines(x_vals, d4, col = "green4", lwd = 2, lty = 1)
# Legend
legend("topright",
```

```

col = c("red", "black", "yellow", "green4"),
lwd = 2,
lty = c(1, 2, 1, 1),
legend = c(expression(paste(sigma, " = 1.4, ", mu, " = 0.7")),
            expression(paste(sigma, " = 1.2, ", mu, " = 0.3")),
            expression(paste(sigma, " = 1.0, ", mu, " = 3.0")),
            expression(paste(sigma, " = 1.0, ", mu, " = 6.0))),
bty = "n")

# Example 2
# Checking if the cumulative curves converge to 1
curve(pGL2(x, mu=0.7, sigma=1.4),
      from=0.00001, to=40,
      ylim=c(0, 1),
      col="royalblue1", lwd=2,
      main="Cumulative Distribution Function",
      xlab="x", ylab="F(x)")
curve(pGL2(x, mu=0.3, sigma=1.2),
      col="tomato",
      lwd=2,
      add=TRUE)
legend("bottomright", legend=c("mu=0.7, sigma=1.4",
                               "mu=0.3, sigma=1.2"),
      col=c("royalblue1", "tomato", "seagreen"), lwd=2, cex=0.5)

# Example 3
# The quantile function
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qGL2(p, mu=3, sigma=1), y=p, xlab="Quantile",
     las=1, ylab="Probability", main="Quantile function ")
curve(pGL2(x, mu=3, sigma=1),
      from=0, add=TRUE, col="tomato", lwd=2.5)

# Example 4
# The random function
set.seed(123)
x <- rGL2(5000, mu=0.7, sigma=1.4)
hist(x, breaks=50, freq=FALSE,
     main="rGL2 vs theory density",
     xlab="x", col="lightblue", border="white")
curve(dGL2(x, mu=0.7, sigma=1.4),
      add=TRUE, col="red", lwd=2)

# Example 5
# The Hazard function
curve(hGL2(x, mu=0.7, sigma=1.4), from=0.001, to=40,
     col="tomato", ylab="Hazard function", las=1)

```

**Description**

Density, distribution function, quantile function, random generation and hazard function for the generalized modified weibull distribution with parameters mu, sigma, nu and tau.

**Usage**

```
dGMW(x, mu, sigma, nu, tau, log = FALSE)
pGMW(q, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)
qGMW(p, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)
rGMW(n, mu, sigma, nu, tau)
hGMW(x, mu, sigma, nu, tau, log = FALSE)
```

**Arguments**

x, q	vector of quantiles.
mu	scale parameter.
sigma	shape parameter.
nu	shape parameter.
tau	acceleration parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

The generalized modified weibull with parameters mu, sigma, nu and tau has density given by

$$f(x) = \mu\sigma x^{\nu-1}(\nu + \tau x) \exp(\tau x - \mu x^{\nu} e^{\tau x}) [1 - \exp(-\mu x^{\nu} e^{\tau x})]^{\sigma-1},$$

for  $x > 0$ .

**Value**

dGMW gives the density, pGMW gives the distribution function, qGMW gives the quantile function, rGMW generates random deviates and hGMW gives the hazard function.

**See Also**

[GMW](#)

**Examples**

```

old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dGMW(x, mu=2, sigma=0.5, nu=2, tau=1.5), from=0, to=0.8,
      ylim=c(0, 2), col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pGMW(x, mu=2, sigma=0.5, nu=2, tau=1.5),
      from=0, to=1.2, col="red", las=1, ylab="F(x)")
curve(pGMW(x, mu=2, sigma=0.5, nu=2, tau=1.5, lower.tail=FALSE),
      from=0, to=1.2, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qGMW(p, mu=2, sigma=0.5, nu=2, tau=1.5), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pGMW(x, mu=2, sigma=0.5, nu=2, tau=1.5), from=0, add=TRUE, col="red")

## The random function
hist(rGMW(n=1000, mu=2, sigma=0.5, nu=2, tau=1.5), freq=FALSE,
     xlab="x", main="", las=1)
curve(dGMW(x, mu=2, sigma=0.5, nu=2, tau=1.5), from=0, add=TRUE, col="red")

## The Hazard function
par(mfrow=c(1,1))
curve(hGMW(x, mu=2, sigma=0.5, nu=2, tau=1.5), from=0, to=1, ylim=c(0, 16),
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters

```

dGWF

*Generalized Weibull distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the generalized Weibull distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```

dGWF(x, mu, sigma, nu, log = FALSE)

pGWF(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

qGWF(p, mu, sigma, nu)

rGWF(n, mu, sigma, nu)

```

```
hGWF(x, mu, sigma, nu)
```

### Arguments

x, q	vector of quantiles.
mu	parameter one.
sigma	parameter two.
nu	parameter three.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

### Details

The generalized Weibull with parameters mu, sigma and nu has density given by

$$f(x) = \mu\sigma x^{\sigma-1} (1 - \mu\nu x^\sigma)^{\frac{1}{\nu}-1}$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $-\infty < \nu < \infty$ .

### Value

dGWF gives the density, pGWF gives the distribution function, qGWF gives the quantile function, rGWF generates random deviates and hGWF gives the hazard function.

### Author(s)

Jaime Mosquera, <jmosquerag@unal.edu.co>

### References

Mudholkar, G. S., & Kollia, G. D. (1994). Generalized Weibull family: a structural analysis. *Communications in statistics-theory and methods*, 23(4), 1149-1171.

### Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(
  dGWF(x, mu = 5, sigma = 2, nu = -0.2),
  from = 0, to = 5, col = "red", las = 1, ylab = "f(x)"
)

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
```

```

curve(
  pGWF(x, mu = 5, sigma = 2, nu = -0.2),
  from = 0, to = 5, ylim = c(0, 1),
  col = "red", las = 1, ylab = "F(x)"
)
curve(
  pGWF(
    x, mu = 5, sigma = 2, nu = -0.2,
    lower.tail = FALSE
  ),
  from = 0, to = 5, ylim = c(0, 1),
  col = "red", las = 1, ylab = "R(x)"
)

## The quantile function
p <- seq(from = 0, to = 0.999, length.out = 100)
plot(
  x = qGWF(p, mu = 5, sigma = 2, nu = -0.2),
  y = p, xlab = "Quantile", las = 1,
  ylab = "Probability"
)
curve(
  pGWF(x, mu = 5, sigma = 2, nu = -0.2),
  from = 0, add = TRUE, col = "red"
)

## The random function
hist(
  rGWF(n = 10000, mu = 5, sigma = 2, nu = -0.2),
  freq = FALSE, xlab = "x", las = 1, main = "", ylim = c(0, 2.0)
)
curve(dGWF(x, mu = 5, sigma = 2, nu = -0.2),
  from = 0, add = TRUE, col = "red"
)

## The Hazard function
par(mfrow = c(1, 1))
curve(
  hGWF(x, mu = 0.003, sigma = 5e-6, nu = 0.025),
  from = 0, to = 250, col = "red",
  ylab = "Hazard function", las = 1
)

par(old_par) # restore previous graphical parameters

```

**Description**

Density, distribution function, quantile function, random generation and hazard function for the inverse weibull distribution with parameters  $\mu$  and  $\sigma$ .

**Usage**

```
dIW(x, mu, sigma, log = FALSE)

pIW(q, mu, sigma, lower.tail = TRUE, log.p = FALSE)

qIW(p, mu, sigma, lower.tail = TRUE, log.p = FALSE)

rIW(n, mu, sigma)

hIW(x, mu, sigma)
```

**Arguments**

x, q	vector of quantiles.
mu	scale parameter.
sigma	shape parameters.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

The inverse weibull distribution with parameters mu and sigma has density given by

$$f(x) = \mu\sigma x^{-\sigma-1} \exp(\mu x^{-\sigma})$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$

**Value**

dIW gives the density, pIW gives the distribution function, qIW gives the quantile function, rIW generates random deviates and hIW gives the hazard function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. Reliability Engineering & System Safety, 124, 32-55.

Drapella, A. (1993). The complementary Weibull distribution: unknown or just forgotten?. Quality and reliability engineering international, 9(4), 383-385.

**See Also**

[IW](#)

**Examples**

```

old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dIW(x, mu=5, sigma=2.5), from=0, to=10,
      ylim=c(0, 0.55), col="red", las=1, ylab="f(x)")
#'
## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pIW(x, mu=5, sigma=2.5),
      from=0, to=10, col="red", las=1, ylab="F(x)")
curve(pIW(x, mu=5, sigma=2.5, lower.tail=FALSE),
      from=0, to=10, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qIW(p, mu=5, sigma=2.5), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pIW(x, mu=5, sigma=2.5), from=0, add=TRUE, col="red")

## The random function
hist(rIW(n=10000, mu=5, sigma=2.5), freq=FALSE, xlim=c(0,60),
     xlab="x", las=1, main="")
curve(dIW(x, mu=5, sigma=2.5), from=0, add=TRUE, col="red")

## The Hazard function
par(mfrow=c(1,1))
curve(hIW(x, mu=5, sigma=2.5), from=0, to=15, ylim=c(0, 0.9),
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters

```

---

dKumIW

*The Kumaraswamy Inverse Weibull distribution*


---

**Description**

Density, distribution function, quantile function, random generation and hazard function for the Kumaraswamy Inverse Weibull distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```

dKumIW(x, mu, sigma, nu, log = FALSE)

pKumIW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

qKumIW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

rKumIW(n, mu, sigma, nu)

```

hKumIW(x, mu, sigma, nu)

### Arguments

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
nu	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

### Details

The Kumaraswamy Inverse Weibull Distribution with parameters mu, sigma and nu has density given by

$$f(x) = \mu\sigma\nu x^{-\mu-1} \exp -\sigma x^{-\mu} (1 - \exp -\sigma x^{-\mu})^{\nu-1},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

### Value

dKumIW gives the density, pKumIW gives the distribution function, qKumIW gives the quantile function, rKumIW generates random deviates and hKumIW gives the hazard function.

### Author(s)

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

### References

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. Reliability Engineering & System Safety, 124, 32-55.

Shahbaz, M. Q., Shahbaz, S., & Butt, N. S. (2012). The Kumaraswamy Inverse Weibull Distribution. Pakistan journal of statistics and operation research, 479-489.

### See Also

[KumIW](#)

**Examples**

```

old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
par(mfrow = c(1, 1))
curve(dKumIW(x, mu = 1.5, sigma= 1.5, nu = 1), from = 0, to = 8.5,
      col = "red", las = 1, ylab = "f(x)")

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pKumIW(x, mu = 1.5, sigma= 1.5, nu = 1), from = 0, to = 8.5,
      ylim = c(0, 1), col = "red", las = 1, ylab = "F(x)")
curve(pKumIW(x, mu = 1.5, sigma= 1.5, nu = 1, lower.tail = FALSE),
      from = 0, to = 6, ylim = c(0, 1), col = "red", las = 1, ylab = "R(x)")

## The quantile function
p <- seq(from = 0, to = 0.99999, length.out = 100)
plot(x = qKumIW(p=p, mu = 1.5, sigma= 1.5, nu = 10), y = p,
     xlab = "Quantile", las = 1, ylab = "Probability")
curve(pKumIW(x, mu = 1.5, sigma= 1.5, nu = 10), from = 0, add = TRUE,
     col = "red")

## The random function
hist(rKumIW(1000, mu = 1.5, sigma= 1.5, nu = 5), freq = FALSE, xlab = "x",
     las = 1, ylim = c(0, 1.5), main = "")
curve(dKumIW(x, mu = 1.5, sigma= 1.5, nu = 5), from = 0, to = 8, add = TRUE,
     col = "red")

## The Hazard function
par(mfrow=c(1,1))
curve(hKumIW(x, mu = 1.5, sigma= 1.5, nu = 1), from = 0, to = 3,
     ylim = c(0, 0.7), col = "red", ylab = "Hazard function", las = 1)

par(old_par) # restore previous graphical parameters

```

dLIN

*Lindley distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Lindley distribution with parameter  $\mu$ .

**Usage**

```
dLIN(x, mu, log = FALSE)
```

```
pLIN(q, mu, lower.tail = TRUE, log.p = FALSE)
```

```
qLIN(p, mu, lower.tail = TRUE, log.p = FALSE)
```

```
rLIN(n, mu)
```

```
hLIN(x, mu, log = FALSE)
```

### Arguments

x, q	vector of quantiles.
mu	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

### Details

Lindley Distribution with parameter mu has density given by

$$f(x) = \frac{\mu^2}{\mu+1}(1+x)\exp(-\mu x),$$

for  $x > 0$  and  $\mu > 0$ . These function were taken form LindleyR package.

### Value

dLIN gives the density, pLIN gives the distribution function, qLIN gives the quantile function, rLIN generates random deviates and hLIN gives the hazard function.

### Author(s)

Freddy Hernandez, <fhernanb@unal.edu.co>

### References

Lindley, D. V. (1958). Fiducial distributions and Bayes' theorem. Journal of the Royal Statistical Society. Series B (Methodological), 102-107.

### See Also

[LIN](#)

### Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dLIN(x, mu=1.5), from=0.0001, to=10,
      col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
```

```

par(mfrow=c(1, 2))
curve(pLIN(x, mu=2), from=0.0001, to=10, col="red", las=1, ylab="F(x)")
curve(pLIN(x, mu=2, lower.tail=FALSE), from=0.0001,
      to=10, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qLIN(p, mu=2), y=p, xlab="Quantile", las=1, ylab="Probability")
curve(pLIN(x, mu=2), from=0, add=TRUE, col="red")

## The random function
hist(rLIN(n=10000, mu=2), freq=FALSE, xlab="x", las=1, main="")
curve(dLIN(x, mu=2), from=0.09, to=5, add=TRUE, col="red")

## The Hazard function
curve(hLIN(x, mu=2), from=0.001, to=10, col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters

```

dLW

*The Log-Weibull distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Log-Weibull distribution with parameters  $\mu$  and  $\sigma$ .

**Usage**

```

dLW(x, mu, sigma, log = FALSE)

pLW(q, mu, sigma, lower.tail = TRUE, log.p = FALSE)

qLW(p, mu, sigma, lower.tail = TRUE, log.p = FALSE)

rLW(n, mu, sigma)

hLW(x, mu, sigma)

```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter.
<code>sigma</code>	parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

**Details**

The Log-Weibull Distribution with parameters  $\mu$  and  $\sigma$  has density given by

$$f(y) = (1/\sigma)e^{((y-\mu)/\sigma)} \exp\{-e^{((y-\mu)/\sigma)}\},$$

for  $-\infty < y < \infty$ .

**Value**

dLW gives the density, pLW gives the distribution function, qLW gives the quantile function, rLW generates random deviates and hLW gives the hazard function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. Reliability Engineering & System Safety, 124, 32-55.

Gumbel, E. J. (1958). Statistics of extremes. Columbia university press.

**See Also**

[LW](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dLW(x, mu=0, sigma=1.5), from=-8, to=5,
      col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pLW(x, mu=0, sigma=1.5),
      from=-8, to=5, col="red", las=1, ylab="F(x)")
curve(pLW(x, mu=0, sigma=1.5, lower.tail=FALSE),
      from=-8, to=5, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qLW(p, mu=0, sigma=1.5), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pLW(x, mu=0, sigma=1.5), from=-8, to=5, add=TRUE, col="red")

## The random function
hist(rLW(n=10000, mu=0, sigma=1.5), freq=FALSE,
     xlab="x", las=1, main="")
curve(dLW(x, mu=0, sigma=1.5), from=-15, to=6, add=TRUE, col="red")
```

```
## The Hazard function
par(mfrow=c(1,1))
curve(hLW(x, mu=0, sigma=1.5), from=-8, to=7,
      col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters
```

dMOEIW

*The Marshall-Olkin Extended Inverse Weibull distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Marshall-Olkin Extended Inverse Weibull distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```
dMOEIW(x, mu, sigma, nu, log = FALSE)

pMOEIW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

qMOEIW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

rMOEIW(n, mu, sigma, nu)

hMOEIW(x, mu, sigma, nu)
```

**Arguments**

$x, q$	vector of quantiles.
$\mu$	parameter.
$\sigma$	parameter.
$\nu$	parameter.
$\log, \log.p$	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
$\text{lower.tail}$	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
$p$	vector of probabilities.
$n$	number of observations.

**Details**

The Marshall-Olkin Extended Inverse Weibull distribution  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(x) = \frac{\mu \sigma \nu x^{-(\sigma+1)} \exp\{-\mu x^{-\sigma}\}}{\{\nu - (\nu-1) \exp\{-\mu x^{-\sigma}\}\}^2},$$

for  $x > 0$ .

**Value**

dMOEIW gives the density, pMOEIW gives the distribution function, qMOEIW gives the quantile function, rMOEIW generates random deviates and hMOEIW gives the hazard function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Okasha, H. M., El-Baz, A. H., Tarabia, A. M. K., & Basheer, A. M. (2017). Extended inverse Weibull distribution with reliability application. *Journal of the Egyptian Mathematical Society*, 25(3), 343-349.

**See Also**

[MOEIW](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dMOEIW(x, mu=0.6, sigma=1.7, nu=0.3), from=0, to=2,
      col="red", ylab="f(x)", las=1)

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pMOEIW(x, mu=0.6, sigma=1.7, nu=0.3),
      from=0.0001, to=2, col="red", las=1, ylab="F(x)")
curve(pMOEIW(x, mu=0.6, sigma=1.7, nu=0.3, lower.tail=FALSE),
      from=0.0001, to=2, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qMOEIW(p, mu=0.6, sigma=1.7, nu=0.3), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pMOEIW(x, mu=0.6, sigma=1.7, nu=0.3),
      from=0, add=TRUE, col="red")

## The random function
hist(rMOEIW(n=1000, mu=0.6, sigma=1.7, nu=0.3), freq=FALSE,
     xlab="x", las=1, main="")
curve(dMOEIW(x, mu=0.6, sigma=1.7, nu=0.3),
      from=0.001, to=4, add=TRUE, col="red")

## The Hazard function
curve(hMOEIW(x, mu=0.5, sigma=0.7, nu=1), from=0.001, to=3,
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters
```

dMOEW

*The Marshall-Olkin Extended Weibull distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Marshall-Olkin Extended Weibull distribution with parameters mu, sigma and nu.

**Usage**

```
dMOEW(x, mu, sigma, nu, log = FALSE)
```

```
pMOEW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
```

```
qMOEW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
```

```
rMOEW(n, mu, sigma, nu)
```

```
hMOEW(x, mu, sigma, nu)
```

**Arguments**

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
nu	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

The Marshall-Olkin Extended Weibull distribution mu, sigma and nu has density given by

$$f(x) = \frac{\mu\sigma\nu(\nu x)^{\sigma-1} \exp\{-(\nu x)^\sigma\}}{\{1-(1-\mu)\exp\{-(\nu x)^\sigma\}\}^2},$$

for  $x > 0$ .

**Value**

dMOEW gives the density, pMOEW gives the distribution function, qMOEW gives the quantile function, rMOEW generates random deviates and hMOEW gives the hazard function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

## References

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Ghitany, M. E., Al-Hussaini, E. K., & Al-Jarallah, R. A. (2005). Marshall–Olkin extended Weibull distribution and its application to censored data. *Journal of Applied Statistics*, 32(10), 1025-1034.

## See Also

[MOEW](#)

## Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dMOEW(x, mu=0.5, sigma=0.7, nu=1), from=0.001, to=1,
      col="red", ylab="f(x)", las=1)

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pMOEW(x, mu=0.5, sigma=0.7, nu=1),
      from=0.0001, to=2, col="red", las=1, ylab="F(x)")
curve(pMOEW(x, mu=0.5, sigma=0.7, nu=1, lower.tail=FALSE),
      from=0.0001, to=2, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qMOEW(p, mu=0.5, sigma=0.7, nu=1), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pMOEW(x, mu=0.5, sigma=0.7, nu=1),
      from=0, add=TRUE, col="red")

## The random function
hist(rMOEW(n=100, mu=0.5, sigma=0.7, nu=1), freq=FALSE,
     xlab="x", ylim=c(0, 1), las=1, main="")
curve(dMOEW(x, mu=0.5, sigma=0.7, nu=1),
      from=0.001, to=2, add=TRUE, col="red")

## The Hazard function
curve(hMOEW(x, mu=0.5, sigma=0.7, nu=1), from=0.001, to=3,
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters
```

**Description**

Density, distribution function, quantile function, random generation and hazard function for the Marshall-Olkin Kappa distribution with parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$ .

**Usage**

```
dMOK(x, mu, sigma, nu, tau, log = FALSE)

pMOK(q, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)

qMOK(p, mu, sigma, nu, tau, lower.tail = TRUE, log.p = FALSE)

rMOK(n, mu, sigma, nu, tau)

hMOK(x, mu, sigma, nu, tau)
```

**Arguments**

$x, q$	vector of quantiles.
$\mu$	parameter.
$\sigma$	parameter.
$\nu$	parameter.
$\tau$	parameter.
$\log, \log.p$	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
$\text{lower.tail}$	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
$p$	vector of probabilities.
$n$	number of observations.

**Details**

The Marshall-Olkin Kappa distribution with parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$  has density given by:

$$f(x) = \frac{\tau \frac{\mu\nu}{\sigma} \left(\frac{x}{\sigma}\right)^{\nu-1} \left(\mu + \left(\frac{x}{\sigma}\right)^{\mu\nu}\right)^{-\frac{\mu+1}{\mu}}}{\left[\tau + (1-\tau) \left(\frac{\left(\frac{x}{\sigma}\right)^{\mu\nu}}{\mu + \left(\frac{x}{\sigma}\right)^{\mu\nu}}\right)^{\frac{1}{\mu}}\right]^2}$$

for  $x > 0$ .

**Value**

dMOK gives the density, pMOK gives the distribution function, qMOK gives the quantile function, rMOK generates random deviates and hMOK gives the hazard function.

**Author(s)**

Angel Muñoz,

## References

Javed, M., Nawaz, T., & Irfan, M. (2019). The Marshall-Olkin kappa distribution: properties and applications. *Journal of King Saud University-Science*, 31(4), 684-691.

## See Also

[MOK](#)

## Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
par(mfrow = c(1,1))
curve(dMOK(x = x, mu = 1, sigma = 3.5, nu = 3, tau = 2), from = 0, to = 15,
      ylab = 'f(x)', col = 2, las = 1)

## The cumulative distribution and the Reliability function

par(mfrow = c(1,2))
curve(pMOK(q = x, mu = 1, sigma = 2.5, nu = 3, tau = 2), from = 0, to = 10,
      col = 2, lwd = 2, las = 1, ylab = 'F(x)')
curve(pMOK(q = x, mu = 1, sigma = 2.5, nu = 3, tau = 2, lower.tail = FALSE), from = 0, to = 10,
      col = 2, lwd = 2, las = 1, ylab = 'R(x)')

## The quantile function
p <- seq(from = 0.00001, to = 0.99999, length.out = 100)
plot(x = qMOK(p = p, mu = 4, sigma = 2.5, nu = 3, tau = 2), y = p, xlab = 'Quantile',
     las = 1, ylab = 'Probability')
curve(pMOK(q = x, mu = 4, sigma = 2.5, nu = 3, tau = 2), from = 0, to = 15,
      add = TRUE, col = 2)

## The random function

hist(rMOK(n = 10000, mu = 1, sigma = 2.5, nu = 3, tau = 2), freq = FALSE,
     xlab = "x", las = 1, main = '', ylim = c(0,.3), xlim = c(0,20), breaks = 50)
curve(dMOK(x, mu = 1, sigma = 2.5, nu = 3, tau = 2), from = 0, to = 15, add = TRUE, col = 2)

## The Hazard function

par(mfrow = c(1,1))
curve(hMOK(x = x, mu = 1, sigma = 2.5, nu = 3, tau = 2), from = 0, to = 20,
      col = 2, ylab = 'Hazard function', las = 1)

par(old_par) # restore previous graphical parameters
```

**Description**

Density, distribution function, quantile function, random generation and hazard function for the modified weibull distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

dMW(x, mu, sigma, nu, log = FALSE)

pMW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

qMW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

rMW(n, mu, sigma, nu)

hMW(x, mu, sigma, nu)

**Arguments**

x, q	vector of quantiles.
mu	shape parameter one.
sigma	parameter two.
nu	scale parameter three.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

The modified weibull distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(x) = \mu(\sigma + \nu x)x^{\sigma-1} \exp(\nu x) \exp(-\mu x^\sigma \exp(\nu x))$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma \geq 0$  and  $\nu \geq 0$ .

**Value**

dMW gives the density, pMW gives the distribution function, qMW gives the quantile function, rMW generates random deviates and hMW gives the hazard function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

- Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. Reliability Engineering & System Safety, 124, 32-55.
- Lai, C. D., Xie, M., & Murthy, D. N. P. (2003). A modified Weibull distribution. IEEE Transactions on reliability, 52(1), 33-37.

**See Also**[MW](#)**Examples**

```

old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dMW(x, mu=2, sigma=1.5, nu=0.2), from=0, to=2,
      ylim=c(0, 1.5), col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pMW(x, mu=2, sigma=1.5, nu=0.2), from=0, to=2,
      col = "red", las=1, ylab="F(x)")
curve(pMW(x, mu=2, sigma=1.5, nu=0.2, lower.tail = FALSE),
      from=0, to=2, col="red", las=1, ylab = "R(x)")

## The quantile function
p <- seq(from=0, to=0.9999, length.out=100)
plot(x=qMW(p, mu=2, sigma=1.5, nu=0.2), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pMW(x, mu=2, sigma=1.5, nu=0.2), from=0, add=TRUE, col="red")

## The random function
hist(rMW(n=1000, mu=2, sigma=1.5, nu=0.2), freq=FALSE,
     xlab="x", las=1, main="")
curve(dMW(x, mu=2, sigma=1.5, nu=0.2), from=0, add=TRUE, col="red")

## The Hazard function
par(mfrow=c(1,1))
curve(hMW(x, mu=2, sigma=1.5, nu=0.2), from=0, to=1.5, ylim=c(0, 5),
     col="red", las=1, ylab="H(x)", las=1)

par(old_par) # restore previous graphical parameters

```

dNEE

*The New Exponentiated Exponential distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the two-parameter New Exponentiated Exponential with parameters  $\mu$  and  $\sigma$ .

**Usage**

```
dNEE(x, mu = 1, sigma = 1, log = FALSE)
```

```
pNEE(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
```

```
qNEE(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
```

```
rNEE(n = 1, mu = 1, sigma = 1)
```

```
hNEE(x, mu, sigma, log = FALSE)
```

### Arguments

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

### Details

The New Exponentiated Exponential distribution with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x|\mu, \sigma) = \log(2^\sigma)\mu \exp(-\mu x)(1 - \exp(-\mu x))^{\sigma-1}2^{(1-\exp(-\mu x))^\sigma},$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ .

Note: In this implementation we changed the original parameters  $\theta$  for  $\mu$  and  $\alpha$  for  $\sigma$ , we did it to implement this distribution within gamlss framework.

### Value

dNEE gives the density, pNEE gives the distribution function, qNEE gives the quantile function, rNEE generates random deviates and hNEE gives the hazard function.

### Author(s)

Juliana Garcia, <juliana.garciav@udea.edu.co>

### References

Hassan, Anwar, I. H. Dar, and M. A. Lone. "A New Class of Probability Distributions With An Application to Engineering Data." *Pakistan Journal of Statistics and Operation Research* 20.2 (2024): 217-231.

### See Also

[NEE](#)

**Examples**

```

# Example 1
# Plotting the mass function for different parameter values
curve(dNEE(x, mu=0.2, sigma=0.3),
      from=0, to=8, col="cadetblue3", las=1, ylab="f(x)")

curve(dNEE(x, mu=1, sigma=4),
      add=TRUE, col="purple")

curve(dNEE(x, mu=1.5, sigma=22),
      add=TRUE, col="goldenrod")

curve(dNEE(x, mu=0.5, sigma=2),
      add=TRUE, col="green3")

legend("topright", col=c("cadetblue3", "purple", "goldenrod", "green3"), lty=1, bty="n",
      legend=c("mu=0.2, sigma=0.3",
              "mu=1.0, sigma=4",
              "mu=1.5, sigma=22",
              "mu=0.5, sigma=2"))

# Example 2
# Checking if the cumulative curves converge to 1
curve(pNEE(x, mu=0.2, sigma=0.3), ylim=c(0, 1),
      from=0, to=8, col="cadetblue3", las=1, ylab="F(x)")

curve(pNEE(x, mu=1, sigma=4),
      add=TRUE, col="purple")

curve(pNEE(x, mu=1.5, sigma=22),
      add=TRUE, col="goldenrod")

curve(pNEE(x, mu=0.5, sigma=2),
      add=TRUE, col="green3")

legend("bottomright", col=c("cadetblue3", "purple", "goldenrod", "green3"), lty=1, bty="n",
      legend=c("mu=0.2, sigma=0.3",
              "mu=1.0, sigma=4",
              "mu=1.5, sigma=22",
              "mu=0.5, sigma=2"))

# Example 3
# Checking the quantile function
mu <- 0.5
sigma <- 2
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qNEE(p, mu=mu, sigma=sigma), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pNEE(x, mu=mu, sigma=sigma), from=0, add=TRUE, col="red")

# Example 4
# Comparing the random generator output with

```

```
# the theoretical probabilities
mu <- 0.5
sigma <- 2
x <- rNEE(n=10000, mu=mu, sigma=sigma)
hist(x, freq=FALSE)
curve(dNEE(x, mu=mu, sigma=sigma), col="tomato", add=TRUE)
```

---

dOW

*The Odd Weibull Distribution*


---

### Description

Density, distribution function, quantile function, random generation and hazard function for the Odd Weibull distribution with parameters mu, sigma and nu.

### Usage

```
dOW(x, mu, sigma, nu, log = FALSE)

pOW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

qOW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

rOW(n, mu, sigma, nu)

hOW(x, mu, sigma, nu)
```

### Arguments

x, q	vector of quantiles.
mu	parameter one.
sigma	parameter two.
nu	parameter three.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[T <= t], otherwise, P[T > t].
p	vector of probabilities.
n	number of observations.

### Details

The Odd Weibull with parameters mu, sigma and nu has density given by

$$f(x) = \left(\frac{\sigma\nu}{x}\right) (\mu x)^\sigma e^{(\mu x)^\sigma} (e^{(\mu x)^\sigma} - 1)^{\nu-1} \left[1 + (e^{(\mu x)^\sigma} - 1)^\nu\right]^{-2}$$

for  $x > 0$ .

**Value**

dOW gives the density, pOW gives the distribution function, qOW gives the quantile function, rOW generates random deviates and hOW gives the hazard function.

**Author(s)**

Jaime Mosquera Gutiérrez <jmosquerag@unal.edu.co>

**References**

Cooray, K. (2006). Generalization of the Weibull distribution: the odd Weibull family. *Statistical Modelling*, 6(3), 265-277.

**See Also**

[OW](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dOW(x, mu=2, sigma=3, nu=0.2), from=0, to=4, ylim=c(0, 2),
      col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pOW(x, mu=2, sigma=3, nu=0.2), from=0, to=4, ylim=c(0, 1),
      col="red", las=1, ylab="F(x)")
curve(pOW(x, mu=2, sigma=3, nu=0.2, lower.tail=FALSE), from=0,
      to=4, ylim=c(0, 1), col="red", las=1,
      ylab = "R(x)")

## The quantile function
p <- seq(from=0, to=0.998, length.out=100)
plot(x = qOW(p, mu=2, sigma=3, nu=0.2), y=p, xlab="Quantile", las=1,
     ylab="Probability")
curve(pOW(x, mu=2, sigma=3, nu=0.2), from=0, add=TRUE, col="red")

## The random function
hist(rOW(n=10000, mu=2, sigma = 3, nu = 0.2), freq=FALSE, ylim = c(0, 2),
     xlab = "x", las = 1, main = "")
curve(dOW(x, mu=2, sigma=3, nu=0.2), from=0, ylim=c(0, 2), add=TRUE,
     col = "red")

## The Hazard function
par(mfrow=c(1,1))
curve(hOW(x, mu=2, sigma=3, nu=0.2), from=0, to=2.5, ylim=c(0, 3),
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters
```

dPL

*The Power Lindley distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Power Lindley distribution with parameters  $\mu$  and  $\sigma$ .

**Usage**

dPL(x, mu, sigma, log = FALSE)

pPL(q, mu, sigma, lower.tail = TRUE, log.p = FALSE)

qPL(p, mu, sigma, lower.tail = TRUE, log.p = FALSE)

rPL(n, mu, sigma)

hPL(x, mu, sigma)

**Arguments**

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
p	vector of probabilities.
n	number of observations.

**Details**

The Power Lindley Distribution with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x) = \frac{\mu\sigma^2}{\sigma+1}(1+x^\mu)x^{\mu-1}\exp(-\sigma x^\mu),$$

for  $x > 0$ .

**Value**

dPL gives the density, pPL gives the distribution function, qPL gives the quantile function, rPL generates random deviates and hPL gives the hazard function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

## References

- Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.
- Ghitany, M. E., Al-Mutairi, D. K., Balakrishnan, N., & Al-Enezi, L. J. (2013). Power Lindley distribution and associated inference. *Computational Statistics & Data Analysis*, 64, 20-33.

## See Also

[PL](#)

## Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dPL(x, mu=1.5, sigma=0.2), from=0.1, to=10,
      col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pPL(x, mu=1.5, sigma=0.2),
      from=0.1, to=10, col="red", las=1, ylab="F(x)")
curve(pPL(x, mu=1.5, sigma=0.2, lower.tail=FALSE),
      from=0.1, to=10, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qPL(p, mu=1.5, sigma=0.2), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pPL(x, mu=1.5, sigma=0.2), from=0.1, add=TRUE, col="red")

## The random function
hist(rPL(n=1000, mu=1.5, sigma=0.2), freq=FALSE,
     xlab="x", las=1, main="")
curve(dPL(x, mu=1.5, sigma=0.2), from=0.1, to=15, add=TRUE, col="red")

## The Hazard function
par(mfrow=c(1,1))
curve(hPL(x, mu=1.5, sigma=0.2), from=0.1, to=15,
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters
```

## Description

Density, distribution function, quantile function, random generation and hazard function for the Quasi XGamma Poisson distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```
dQXGP(x, mu, sigma, nu, log = FALSE)
pQXGP(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
qQXGP(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
rQXGP(n, mu, sigma, nu)
hQXGP(x, mu, sigma, nu)
```

**Arguments**

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
nu	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

The Quasi XGamma Poisson distribution with parameters mu, sigma and nu has density given by:

$$f(x) = K(\mu, \sigma, \nu) \left( \frac{\sigma^2 x^2}{2} + \mu \right) \exp\left( \frac{\nu \exp(-\sigma x) (1 + \mu + \sigma x + \frac{\sigma^2 x^2}{2})}{1 + \mu} - \sigma x \right),$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$ ,  $\nu > 1$ .

where

$$K(\mu, \sigma, \nu) = \frac{\nu \sigma}{(\exp(\nu) - 1)(1 + \mu)}$$

**Value**

dQXGP gives the density, pQXGP gives the distribution function, qQXGP gives the quantile function, rQXGP generates random deviates and hQXGP gives the hazard function.

**Author(s)**

Simon Zapata

**References**

Sen, S., Korkmaz, M. Ç., & Yousof, H. M. (2018). The quasi XGamma-Poisson distribution: properties and application. *Istatistik Journal of The Turkish Statistical Association*, 11(3), 65-76.

**See Also**[QXGP](#)**Examples**

```

old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dQXGP(x, mu=0.5, sigma=1, nu=1), from=0.1, to=8,
      ylim=c(0, 0.6), col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pQXGP(x, mu=0.5, sigma=1, nu=1),
      from=0.1, to=8, col="red", las=1, ylab="F(x)")
curve(pQXGP(x, mu=0.5, sigma=1, nu=1, lower.tail=FALSE),
      from=0.1, to=8, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qQXGP(p, mu=0.5, sigma=1, nu=1), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pQXGP(x, mu=0.5, sigma=1, nu=1),
      from=0.1, add=TRUE, col="red")

## The random function
hist(rQXGP(n=1000, mu=0.5, sigma=1, nu=1), freq=FALSE,
     xlab="x", ylim=c(0, 0.4), las=1, main="", xlim=c(0, 15))
curve(dQXGP(x, mu=0.5, sigma=1, nu=1),
      from=0.001, to=500, add=TRUE, col="red")

## The Hazard function
curve(hQXGP(x, mu=0.5, sigma=1, nu=1), from=0.01, to=3,
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters

```

dRNMW

*The Reduced New Modified Weibull distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the reduced new modified Weibull distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```
dRNMW(x, mu, sigma, nu, log = FALSE)
```

```
pRNMW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
```

```
qRNMW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
```

```
rRNMW(n, mu, sigma, nu)
```

```
hRNMW(x, mu, sigma, nu)
```

### Arguments

x, q	vector of quantiles.
mu	parameter one.
sigma	parameter two.
nu	parameter three.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[T <= t], otherwise, P[T > t].
p	vector of probabilities.
n	number of observations.

### Details

The reduced new modified Weibull with parameters mu, sigma and nu has density given by

$$f(x) = \frac{1}{2\sqrt{x}} (\mu + \sigma(1 + 2\nu x)e^{\nu x}) e^{-\mu\sqrt{x} - \sigma\sqrt{x}e^{\nu x}}$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

### Value

dRNMW gives the density, pRNMW gives the distribution function, qRNMW gives the quantile function, rRNMW generates random deviates and hRNMW gives the hazard function.

### Author(s)

Jaime Mosquera, <jmosquera@unal.edu.co>

### References

Almalki, S. J. (2018). A reduced new modified Weibull distribution. Communications in Statistics-Theory and Methods, 47(10), 2297-2313.

**Examples**

```

old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(
  dRNMW(x, mu = 0.05, sigma = 0.00025, nu = 2.2),
  from = 0, to = 5, col = "red", las = 1, ylab = "f(x)"
)

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(
  pRNMW(x, mu = 0.05, sigma = 0.00025, nu = 2.2),
  from = 0, to = 5, ylim = c(0, 1),
  col = "red", las = 1, ylab = "F(x)"
)
curve(
  pRNMW(
    x, mu = 0.05, sigma = 0.00025, nu = 2.2,
    lower.tail = FALSE
  ),
  from = 0, to = 5, ylim = c(0, 1),
  col = "red", las = 1, ylab = "R(x)"
)

## The quantile function
p <- seq(from = 0, to = 0.999, length.out = 100)
plot(
  x = qRNMW(p, mu = 0.05, sigma = 0.00025, nu = 2.2),
  y = p, xlab = "Quantile", las = 1,
  ylab = "Probability"
)
curve(
  pRNMW(x, mu = 0.05, sigma = 0.00025, nu = 2.2),
  from = 0, add = TRUE, col = "red"
)

## The random function
hist(
  rRNMW(n = 10000, mu = 0.05, sigma = 0.00025, nu = 2.2),
  freq = FALSE, xlab = "x", las = 1, main = "", ylim = c(0,0.8)
)
curve(dRNMW(x, mu = 0.05, sigma = 0.00025, nu = 2.2),
  from = 0, add = TRUE, col = "red"
)

## The Hazard function
par(mfrow = c(1, 1))
curve(
  hRNMW(x, mu = 0.003, sigma = 5e-6, nu = 0.025),
  from = 0, to = 250, col = "red",
  ylab = "Hazard function", las = 1
)

```

```
)
par(old_par) # restore previous graphical parameters
```

---

dRW

*The Reflected Weibull distribution*


---

### Description

Density, distribution function, quantile function, random generation and hazard function for the Reflected Weibull Distribution with parameters  $\mu$  and  $\sigma$ .

### Usage

```
dRW(x, mu, sigma, log = FALSE)
pRW(q, mu, sigma, lower.tail = TRUE, log.p = FALSE)
qRW(p, mu, sigma, lower.tail = TRUE, log.p = FALSE)
rRW(n, mu, sigma)
hRW(x, mu, sigma)
```

### Arguments

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter.
<code>sigma</code>	parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

### Details

The Reflected Weibull Distribution with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x) = \mu\sigma(-x)^{\sigma-1}e^{-\mu(-x)^\sigma},$$

for  $x < 0$ .

### Value

dRW gives the density, pRW gives the distribution function, qRW gives the quantile function, rRW generates random deviates and hRW gives the hazard function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. Reliability Engineering & System Safety, 124, 32-55.

Cohen, A. C. (1973). The reflected Weibull distribution. Technometrics, 15(4), 867-873.

**See Also**

[RW](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dRW(x, mu=1, sigma=1), from=-5, to=-0.01,
      col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pRW(x, mu=1, sigma=1),
      from=-5, to=-0.01, col="red", las=1, ylab="F(x)")
curve(pRW(x, mu=1, sigma=1, lower.tail=FALSE),
      from=-5, to=-0.01, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qRW(p, mu=1, sigma=1), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pRW(x, mu=1, sigma=1), from=-5, add=TRUE, col="red")

## The random function
hist(rRW(n=10000, mu=1, sigma=1), freq=FALSE,
     xlab="x", las=1, main="")
curve(dRW(x, mu=1, sigma=1), from=-5, to=-0.01, add=TRUE, col="red")

## The Hazard function
par(mfrow=c(1,1))
curve(hRW(x, mu=1, sigma=1), from=-0.3, to=-0.01,
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters
```

dSZMW

*The Sarhan and Zaindin's Modified Weibull distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for Sarhan and Zaindin's modified weibull distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```
dSZMW(x, mu, sigma, nu, log = FALSE)
pSZMW(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
qSZMW(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
rSZMW(n, mu, sigma, nu)
hSZMW(x, mu, sigma, nu)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>mu</code>	scale parameter.
<code>sigma</code>	shape parameter.
<code>nu</code>	shape parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

**Details**

The Sarhan and Zaindin's modified weibull with parameters  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(x) = (\mu + \sigma \nu x^{\nu-1}) \exp(-\mu x - \sigma x^\nu)$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

**Value**

dSZMW gives the density, pSZMW gives the distribution function, qSZMW gives the quantile function, rSZMW generates random deviates and hSZMW gives the hazard function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

## References

- Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.
- Sarhan, A. M., & Zaindin, M. (2009). Modified Weibull distribution. *APPS. Applied Sciences*, 11, 123-136.

## See Also

[SZMW](#)

## Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dSZMW(x, mu = 2, sigma = 1.5, nu = 0.2), from = 0, to = 2,
      ylim = c(0, 1.7), col = "red", las = 1, ylab = "f(x)")
## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pSZMW(x, mu = 2, sigma = 1.5, nu = 0.2), from = 0, to = 2, ylim = c(0, 1),
      col = "red", las = 1, ylab = "F(x)")
curve(pSZMW(x, mu = 2, sigma = 1.5, nu = 0.2, lower.tail = FALSE), from = 0,
      to = 2, ylim = c(0, 1), col = "red", las = 1, ylab = "R(x)")

## The quantile function
p <- seq(from = 0, to = 0.99999, length.out = 100)
plot(x = qSZMW(p = p, mu = 2, sigma = 1.5, nu = 0.2), y = p, xlab = "Quantile",
     las = 1, ylab = "Probability")
curve(pSZMW(x, mu = 2, sigma = 1.5, nu = 0.2), from = 0, add = TRUE, col = "red")

## The random function
hist(rSZMW(n = 1000, mu = 2, sigma = 1.5, nu = 0.2), freq = FALSE, xlab = "x",
     las = 1, main = "")
curve(dSZMW(x, mu = 2, sigma = 1.5, nu = 0.2), from = 0, add = TRUE, col = "red")

## The Hazard function
par(mfrow=c(1,1))
curve(hSZMW(x, mu = 2, sigma = 1.5, nu = 0.2), from = 0, to = 3, ylim = c(0, 8),
     col = "red", ylab = "Hazard function", las = 1)

par(old_par) # restore previous graphical parameters
```

---

dWALD

*The Wald distribution*

---

## Description

These functions define the density, distribution function, quantile function and random generation for the Wald distribution with parameter  $\mu$  and  $\sigma$ .

**Usage**

dWALD(x, mu, sigma, log = FALSE)

pWALD(q, mu, sigma, lower.tail = TRUE, log.p = FALSE)

qWALD(p, mu, sigma, lower.tail = TRUE, log.p = FALSE)

rWALD(n, mu, sigma)

**Arguments**

x, q	vector of (non-negative integer) quantiles.
mu	vector of the mu parameter.
sigma	vector of the sigma parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of random values to return.

**Details**

The Wald distribution with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x|\mu, \sigma) = \frac{\sigma}{\sqrt{2\pi x^3}} \exp\left[-\frac{(\sigma - \mu x)^2}{2x}\right],$$

for  $x < 0$ .

**Value**

dWALD gives the density, pWALD gives the distribution function, qWALD gives the quantile function, rWALD generates random deviates.

**Author(s)**

Sofia Cuartas, <scuartas@unal.edu.co>

**References**

Heathcote, A. (2004). Fitting Wald and ex-Wald distributions to response time data: An example using functions for the S-PLUS package. Behavior Research Methods, Instruments, & Computers, 36, 678-694.

**See Also**

[WALD](#)

[WALD](#).

**Examples**

```

# Example 1
# Plotting the mass function for different parameter values
curve(dWALD(x, mu=1, sigma=1),
      from=0, to=3, col="cadetblue3", las=1, ylab="f(x)")

curve(dWALD(x, mu=1, sigma=2),
      add=TRUE, col= "purple")

curve(dWALD(x, mu=2, sigma=4),
      add=TRUE, col="goldenrod")

legend("topright", col=c("cadetblue3", "purple", "goldenrod"),
      lty=1, bty="n",
      legend=c("mu=1, sigma=1",
              "mu=1, sigma=2",
              "mu=2, sigma=4"))

# Example 2
# Checking if the cumulative curves converge to 1
curve(pWALD(x, mu=1, sigma=1), ylim=c(0, 1),
      from=0, to=5, col="cadetblue3", las=1, ylab="F(x)")

curve(pWALD(x, mu=1, sigma=2),
      add=TRUE, col= "purple")

curve(pWALD(x, mu=2, sigma=4),
      add=TRUE, col="goldenrod")

legend("bottomright", col=c("cadetblue3", "purple", "goldenrod"),
      lty=1, bty="n",
      legend=c("mu=1, sigma=1",
              "mu=1, sigma=2",
              "mu=2, sigma=4"))

# Example 3
# Checking the quantile function
mu <- 1
sigma <- 2
p <- seq(from=0, to=0.999, length.out=100)
plot(x=qWALD(p, mu=mu, sigma=sigma), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pWALD(x, mu=mu, sigma=sigma), from=0, add=TRUE, col="red")

# Example 4
# Comparing the random generator output with
# the theoretical probabilities
mu <- 1
sigma <- 20
x <- rWALD(n=10000, mu=mu, sigma=sigma)
hist(x, freq=FALSE)
curve(dWALD(x, mu=mu, sigma=sigma), col="tomato", add=TRUE)

```

dWG

*The Weibull Geometric distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the weibull geometric distribution with parameters mu, sigma and nu.

**Usage**

```
dWG(x, mu, sigma, nu, log = FALSE)
```

```
pWG(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
```

```
qWG(p, sigma, mu, nu, lower.tail = TRUE, log.p = FALSE)
```

```
rWG(n, mu, sigma, nu)
```

```
hWG(x, mu, sigma, nu)
```

**Arguments**

x, q	vector of quantiles.
mu	scale parameter.
sigma	shape parameter.
nu	parameter of geometric random variable.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

The Weibull geometric distribution with parameters mu, sigma and nu has density given by

$$f(x) = (\sigma\mu^\sigma(1-\nu)x^{\sigma-1}\exp(-(\mu x)^\sigma))(1-\nu\exp(-(\mu x)^\sigma))^{-2},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $0 < \nu < 1$ .

**Value**

dWG gives the density, pWG gives the distribution function, qWG gives the quantile function, rWG generates random deviates and hWG gives the hazard function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

Barreto-Souza, W., de Morais, A. L., & Cordeiro, G. M. (2011). The Weibull-geometric distribution. *Journal of Statistical Computation and Simulation*, 81(5), 645-657.

**See Also**

[WG](#)

**Examples**

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dWG(x, mu = 0.9, sigma = 2, nu = 0.5), from = 0, to = 3,
      ylim = c(0, 1.1), col = "red", las = 1, ylab = "f(x)")

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pWG(x, mu = 0.9, sigma = 2, nu = 0.5), from = 0, to = 3,
      ylim = c(0, 1), col = "red", las = 1, ylab = "F(x)")
curve(pWG(x, mu = 0.9, sigma = 2, nu = 0.5, lower.tail = FALSE),
      from = 0, to = 3, ylim = c(0, 1), col = "red", las = 1, ylab = "R(x)")

## The quantile function
p <- seq(from = 0, to = 0.99999, length.out = 100)
plot(x = qWG(p = p, mu = 0.9, sigma = 2, nu = 0.5), y = p,
     xlab = "Quantile", las = 1, ylab = "Probability")
curve(pWG(x, mu = 0.9, sigma = 2, nu = 0.5), from = 0, add = TRUE,
      col = "red")

## The random function
hist(rWG(1000, mu = 0.9, sigma = 2, nu = 0.5), freq = FALSE, xlab = "x",
     ylim = c(0, 1.8), las = 1, main = "")
curve(dWG(x, mu = 0.9, sigma = 2, nu = 0.5), from = 0, add = TRUE,
      col = "red", ylim = c(0, 1.8))

## The Hazard function(
par(mfrow=c(1,1))
curve(hWG(x, mu = 0.9, sigma = 2, nu = 0.5), from = 0, to = 8,
      ylim = c(0, 12), col = "red", ylab = "Hazard function", las = 1)

par(old_par) # restore previous graphical parameters
```

dWGEE

*The Weighted Generalized Exponential-Exponential distribution***Description**

Density, distribution function, quantile function, random generation and hazard function for the Weighted Generalized Exponential-Exponential distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```
dWGEE(x, mu, sigma, nu, log = FALSE)
pWGEE(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
qWGEE(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)
rWGEE(n, mu, sigma, nu)
hWGEE(x, mu, sigma, nu)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>mu</code>	parameter.
<code>sigma</code>	parameter.
<code>nu</code>	parameter.
<code>log, log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.

**Details**

The Weighted Generalized Exponential-Exponential Distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(x) = \sigma\nu \exp(-\nu x)(1 - \exp(-\nu x))^{\sigma-1}(1 - \exp(-\mu\nu x))/1 - \sigma B(\mu + 1, \sigma),$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

**Value**

dWGEE gives the density, pWGEE gives the distribution function, qWGEE gives the quantile function, rWGEE generates random deviates and hWGEE gives the hazard function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

## References

Mahdavi, A. (2015). Two weighted distributions generated by exponential distribution. *Journal of Mathematical Extension*, 9, 1-12.

## See Also

[WGEE](#)

## Examples

```
old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dWGEE(x, mu = 5, sigma = 0.5, nu = 1), from = 0, to = 6,
      ylim = c(0, 1), col = "red", las = 1, ylab = "The probability density function")

## The cumulative distribution and the Reliability function
par(mfrow = c(1, 2))
curve(pWGEE(x, mu = 5, sigma = 0.5, nu = 1), from = 0, to = 6,
      ylim = c(0, 1), col = "red", las = 1, ylab = "The cumulative distribution function")
curve(pWGEE(x, mu = 5, sigma = 0.5, nu = 1, lower.tail = FALSE),
      from = 0, to = 6, ylim = c(0, 1), col = "red", las = 1, ylab = "The Reliability function")

## The quantile function
p <- seq(from = 0, to = 0.99999, length.out = 100)
plot(x = qWGEE(p = p, mu = 5, sigma = 0.5, nu = 1), y = p,
     xlab = "Quantile", las = 1, ylab = "Probability")
curve(pWGEE(x, mu = 5, sigma = 0.5, nu = 1), from = 0, add = TRUE,
      col = "red")

## The random function
hist(rWGEE(1000, mu = 5, sigma = 0.5, nu = 1), freq = FALSE, xlab = "x",
     ylim = c(0, 1), las = 1, main = "")
curve(dWGEE(x, mu = 5, sigma = 0.5, nu = 1), from = 0, add = TRUE,
      col = "red", ylim = c(0, 1))

## The Hazard function(
par(mfrow=c(1,1))
curve(hWGEE(x, mu = 5, sigma = 0.5, nu = 1), from = 0, to = 6,
      ylim = c(0, 1.4), col = "red", ylab = "The hazard function", las = 1)

par(old_par) # restore previous graphical parameters
```

## Description

Density, distribution function, quantile function, random generation and hazard function for the Weibull Poisson distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$ .

**Usage**

```
dWP(x, mu, sigma, nu, log = FALSE)

pWP(q, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

qWP(p, mu, sigma, nu, lower.tail = TRUE, log.p = FALSE)

rWP(n, mu, sigma, nu)

hWP(x, mu, sigma, nu)
```

**Arguments**

x, q	vector of quantiles.
mu	parameter.
sigma	parameter.
nu	parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].
p	vector of probabilities.
n	number of observations.

**Details**

The Weibull Poisson distribution with parameters mu, sigma and nu has density given by

$$f(x) = \frac{\mu\sigma\nu e^{-\nu}}{1-e^{-\nu}} x^{\mu-1} \exp(-\sigma x^{\mu} + \nu \exp(-\sigma x^{\mu})),$$

for  $x > 0$ .

**Value**

dWP gives the density, pWP gives the distribution function, qWP gives the quantile function, rWP generates random deviates and hWP gives the hazard function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Lu, Wanbo, and Daimin Shi. "A new compounding life distribution: the Weibull–Poisson distribution." *Journal of applied statistics* 39.1 (2012): 21-38.

**See Also**

[WP](#)

**Examples**

```

old_par <- par(mfrow = c(1, 1)) # save previous graphical parameters

## The probability density function
curve(dWP(x, mu=1.5, sigma=0.5, nu=10), from=0.0001, to=2,
      col="red", las=1, ylab="f(x)")

## The cumulative distribution and the Reliability function
par(mfrow=c(1, 2))
curve(pWP(x, mu=1.5, sigma=0.5, nu=10),
      from=0.0001, to=2, col="red", las=1, ylab="F(x)")
curve(pWP(x, mu=1.5, sigma=0.5, nu=10, lower.tail=FALSE),
      from=0.0001, to=2, col="red", las=1, ylab="R(x)")

## The quantile function
p <- seq(from=0, to=0.99999, length.out=100)
plot(x=qWP(p, mu=1.5, sigma=0.5, nu=10), y=p, xlab="Quantile",
     las=1, ylab="Probability")
curve(pWP(x, mu=1.5, sigma=0.5, nu=10),
      from=0, add=TRUE, col="red")

## The random function
hist(rWP(n=10000, mu=1.5, sigma=0.5, nu=10), freq=FALSE,
     xlab="x", ylim=c(0, 2.2), las=1, main="")
curve(dWP(x, mu=1.5, sigma=0.5, nu=10),
      from=0.0001, to=4, add=TRUE, col="red")

## The Hazard function
curve(hWP(x, mu=1.5, sigma=0.5, nu=10), from=0.0001, to=5,
     col="red", ylab="Hazard function", las=1)

par(old_par) # restore previous graphical parameters

```

EEG

*The Extended Exponential Geometric family***Description**

The Extended Exponential Geometric family

**Usage**

```
EEG(mu.link = "log", sigma.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.

**Details**

The Extended Exponential Geometric distribution with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x) = \mu\sigma \exp(-\mu x)(1 - (1 - \sigma) \exp(-\mu x))^{-2},$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a EEG distribution in the `gamlss()` function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Adamidis, K., Dimitrakopoulou, T., & Loukas, S. (2005). On an extension of the exponential-geometric distribution. *Statistics & probability letters*, 73(3), 259-269.

**See Also**

[dEEG](#)

**Examples**

```
# Generating some random values with
# known mu, sigma, nu and tau
y <- rEEG(n=100, mu = 1, sigma =1.5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, family=EEG,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma, nu and tau
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n, min=0.1, max=0.2)
x2 <- runif(n, min=0.1, max=0.15)
mu <- exp(0.75 - x1)
sigma <- exp(0.5 - x2)
x <- rEEG(n=n, mu, sigma)
```

```
mod <- gamlss(x~x1, sigma.fo=~x2, family=EGG,
             control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
```

EGG

*The four parameter Exponentiated Generalized Gamma family***Description**

The four parameter Exponentiated Generalized Gamma distribution

**Usage**

```
EGG(mu.link = "log", sigma.link = "log", nu.link = "log", tau.link = "log")
```

**Arguments**

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the <code>mu</code> parameter.
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the <code>sigma</code> .
<code>nu.link</code>	defines the <code>nu.link</code> , with "log" link as the default for the <code>nu</code> parameter.
<code>tau.link</code>	defines the <code>tau.link</code> , with "log" link as the default for the <code>tau</code> parameter.

**Details**

Four parameter Exponentiated Generalized Gamma distribution with parameters `mu`, `sigma`, `nu` and `tau` has density given by

$$f(x) = \frac{\nu\sigma}{\mu\Gamma(\tau)} \left(\frac{x}{\mu}\right)^{\sigma\tau-1} \exp\left\{-\left(\frac{x}{\mu}\right)^\sigma\right\} \left\{\gamma_1\left(\tau, \left(\frac{x}{\mu}\right)^\sigma\right)\right\}^{\nu-1},$$

for  $x > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a EGG distribution in the `gamlss()` function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Cordeiro, G. M., Ortega, E. M., & Silva, G. O. (2011). The exponentiated generalized gamma distribution with application to lifetime data. *Journal of statistical computation and simulation*, 81(7), 827-842.

**See Also**[dEGG](#)**Examples**

```

# Example 1
# Generating some random values with
# known mu, sigma, nu and tau

set.seed(123456)
y <- rEGG(n=100, mu=0.1, sigma=0.8, nu=10, tau=1.5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, tau.fo=~1,
              family=EGG,
              control=gamlss.control(n.cyc=500, trace=FALSE))

# Extracting the fitted values for mu, sigma, nu and tau
# using the inverse link function
exp(coef(mod, what="mu"))
exp(coef(mod, what="sigma"))
exp(coef(mod, what="nu"))
exp(coef(mod, what="tau"))

# Example 2
# Generating random values under some model

# A function to simulate a data set with  $Y \sim \text{EGG}$ 
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu   <- exp(-0.8 + -3 * x1)
  sigma <- exp(0.77 - 2 * x2)
  nu   <- 10
  tau  <- 1.5
  y <- rEGG(n=n, mu=mu, sigma=sigma, nu=nu, tau)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(12345)
dat <- gendat(n=200)

mod <- gamlss(y~x1, sigma.fo=~x2, nu.fo=~1, tau.fo=~1,
              family=EGG, data=dat,
              control=gamlss.control(n.cyc=500, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")

```

```
exp(coef(mod, what="nu"))
exp(coef(mod, what="tau"))
```

EMWEx

*The Exponentiated Modified Weibull Extension family***Description**

The Exponentiated Modified Weibull Extension family

**Usage**

```
EMWEx(mu.link = "log", sigma.link = "log", nu.link = "log", tau.link = "log")
```

**Arguments**

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the mu parameter.
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the sigma.
<code>nu.link</code>	defines the <code>nu.link</code> , with "log" link as the default for the nu parameter.
<code>tau.link</code>	defines the <code>tau.link</code> , with "log" link as the default for the tau parameter.

**Details**

The Beta-Weibull distribution with parameters `mu`, `sigma`, `nu` and `tau` has density given by

$$f(x) = \nu\sigma\tau\left(\frac{x}{\mu}\right)^{\sigma-1} \exp\left(\left(\frac{x}{\mu}\right)^{\sigma} + \nu\mu(1 - \exp\left(\left(\frac{x}{\mu}\right)^{\sigma}\right))\right)(1 - \exp(\nu\mu(1 - \exp\left(\left(\frac{x}{\mu}\right)^{\sigma}\right))))^{\tau-1},$$

for  $x > 0$ ,  $\nu > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\tau > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a EMWEx distribution in the `gamlss()` function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Sarhan, A. M., & Apaloo, J. (2013). Exponentiated modified Weibull extension distribution. *Reliability Engineering & System Safety*, 112, 137-144.

**See Also**

[dEMWEx](#)

**Examples**

```

# Example 1
# Generating some random values with
# known mu, sigma, nu and tau
y <- rEMWEx(n=100, mu = 1, sigma =1.21, nu=1, tau=2)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, tau.fo=~1, family=EMWEx,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma, nu and tau
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))
exp(coef(mod, what='tau'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(0.75 - x1)
sigma <- exp(0.5 - x2)
nu <- 1
tau <- 2
x <- rEMWEx(n=n, mu, sigma, nu, tau)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, tau.fo=~1, family=EMWEx,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))
exp(coef(mod, what="tau"))

```

---

 EOFNH

---

*The Extended Odd Frechet-Nadarajah-Haghighi family*


---

**Description**

The Extended Odd Frechet-Nadarjad-Hanhighi family

**Usage**

```
EOFNH(mu.link = "log", sigma.link = "log", nu.link = "log", tau.link = "log")
```

**Arguments**

mu.link	defines the mu.link, with "log" link as the default for the mu parameter.
sigma.link	defines the sigma.link, with "log" link as the default for the sigma.
nu.link	defines the nu.link, with "log" link as the default for the nu parameter.
tau.link	defines the tau.link, with "log" link as the default for the tau parameter.

**Details**

The Extended Odd Fréchet-Nadarajah-Haghighi distribution with parameters mu, sigma, nu and tau has density given by

$$f(x) = \frac{\mu\sigma\nu\tau(1+\nu x)^{\sigma-1}e^{(1-(1+\nu x)^\sigma)}[1-(1-e^{(1-(1+\nu x)^\sigma)})^\mu]^{\tau-1}}{(1-e^{(1-(1+\nu x)^\sigma)})^{\mu\tau+1}}e^{-[(1-e^{(1-(1+\nu x)^\sigma)})^{-\mu}-1]^\tau},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$ ,  $\nu > 0$  and  $\tau > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a EOFNH distribution in the `gamlss()` function.

**Author(s)**

Helber Santiago Padilla, <hspadillar@unal.edu.co>

**References**

Nasiru, S. (2018). Extended Odd Fréchet-G Family of Distributions Journal of Probability and Statistics, 2018(1), 2931326.

**See Also**

[dEOFNH](#)

**Examples**

```
# Example 1
# Generating some random values with
# known mu, sigma, nu and tau
set.seed(123)
y <- rEOFNH(n=100, mu=1, sigma=2.1, nu=0.8, tau=1)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, tau.fo=~1, family=EOFNH,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma, nu and tau
# using the inverse link function
exp(coef(mod, what="mu"))
exp(coef(mod, what="sigma"))
```

```

exp(coef(mod, what="nu"))
exp(coef(mod, what="tau"))

# Example 2
# Generating random values under the model
n <- 100
x1 <- runif(n)
x2 <- runif(n)
mu <- exp(0.5 - 1.2 * x1)
sigma <- 2.1
nu <- 0.8
tau <- 1
y <- rEOFNH(n=n, mu, sigma, nu, tau)

mod <- gamlss(y~x1, sigma.fo=~1, nu.fo=~1, tau.fo=~1, family=EOFNH,
             control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
exp(coef(mod, what="sigma"))
exp(coef(mod, what="nu"))
exp(coef(mod, what="tau"))

```

---

equipment

*Electronic equipment data*

---

### Description

Time to failure in hours of 18 units of the same electronic device.

### Usage

```
data(equipment)
```

### Format

A vector with 18 observations.

### Examples

```
data(equipment)
hist(equipment, main="", xlab="Time (h)")
```

---

estim_mu_sigma_GL2	estim_mu_sigma_GL2
--------------------	--------------------

---

**Description**

This function generates initial values for the GL2 distribution

**Usage**

```
estim_mu_sigma_GL2(y)
```

**Arguments**

`y` vector with the random sample

**Examples**

```
y <- rGL2(n = 100, mu = 3, sigma = 1.2)
estim_mu_sigma_GL2(y = y)
```

---

EW	<i>The Exponentiated Weibull family</i>
----	---

---

**Description**

The Exponentiated Weibull distribution

**Usage**

```
EW(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

`mu.link` defines the mu.link, with "log" link as the default for the mu parameter.

`sigma.link` defines the sigma.link, with "log" link as the default for the sigma.

`nu.link` defines the nu.link, with "log" link as the default for the nu parameter.

**Details**

The Exponentiated Weibull Distribution with parameters mu, sigma and nu has density given by

$$f(x) = \nu\mu\sigma x^{\sigma-1} \exp(-\mu x^\sigma)(1 - \exp(-\mu x^\sigma))^{\nu-1},$$

for  $x > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a EW distribution in the `gamlss()` function.

**See Also**[dEW](#)**Examples**

```

# Example 1
# Generating some random values with
# known mu, sigma and nu
# Will not be run this example because high number of cycles
# is needed in order to get good estimates
## Not run:
y <- rEW(n=100, mu=2, sigma=1.5, nu=0.5)

# Fitting the model
require(gamlss)
mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='EW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))

## End(Not run)

# Example 2
# Generating random values under some model
# Will not be run this example because high number of cycles
# is needed in order to get good estimates
## Not run:
n <- 200
x1 <- rpois(n, lambda=2)
x2 <- runif(n)
mu <- exp(2 + -3 * x1)
sigma <- exp(3 - 2 * x2)
nu <- 2
x <- rEW(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, family=EW,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))

## End(Not run)

```

**Description**

The function `EXL()` defines The exponentiated XLindley, a two parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

**Usage**

```
EXL(mu.link = "log", sigma.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.

**Details**

The exponentiated XLindley with parameters `mu` and `sigma` has density given by

$$f(x) = \frac{\sigma \mu^2 (2 + \mu + x) \exp(-\mu x)}{(1 + \mu)^2} \left[ 1 - \left( 1 + \frac{\mu x}{(1 + \mu)^2} \right) \exp(-\mu x) \right]^{\sigma - 1}$$

for  $x \geq 0$ ,  $\mu \geq 0$  and  $\sigma \geq 0$ .

Note: In this implementation we changed the original parameters  $\delta$  for  $\mu$  and  $\alpha$  for  $\sigma$  we did it to implement this distribution within `gamlss` framework.

**Value**

Returns a `gamlss.family` object which can be used to fit a EXL distribution in the `gamlss()` function.

**Author(s)**

Manuel Gutierrez Tangarife, <mgutierrez@unal.edu.co>

**References**

Alomair, A. M., Ahmed, M., Tariq, S., Ahsan-ul-Haq, M., & Talib, J. (2024). An exponentiated XLindley distribution with properties, inference and applications. *Heliyon*, 10(3).

**See Also**

[EXL](#).

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
y <- rEXL(n=300, mu=0.75, sigma=1.3)

# Fitting the model
require(gamlss)
```

```

mod1 <- gamlss(y~1, sigma.fo=~1, family=EXL,
               control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y ~ EXL
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(1.45 - 3 * x1)
  sigma <- exp(2 - 1.5 * x2)
  y <- rEXL(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(1234)
dat <- gendat(n=100)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
               family=EXL, data=dat,
               control=gamlss.control(n.cyc=5000, trace=FALSE))

summary(mod2)

# Example 3
# Mortality rate due to COVID-19 for 30 days (31st March to April 30, 2020)
# recorded for the Netherlands.
# Taken from Alomair et al. (2024) page 12.

x <- c(14.918, 10.656, 12.274, 10.289, 10.832, 7.099, 5.928, 13.211,
       7.968, 7.584, 5.555, 6.027, 4.097, 3.611, 4.960, 7.498, 6.940,
       5.307, 5.048, 2.857, 2.254, 5.431, 4.462, 3.883,
       3.461, 3.647, 1.974, 1.273, 1.416, 4.235)

mod3 <- gamlss(x~1, sigma.fo=~1, family=EXL,
               control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod3, what="mu"))
exp(coef(mod3, what="sigma"))

# Replicating figure 4 from Alomair et al. (2024)
# Hist and estimated pdf
hist(x, freq=FALSE)
curve(dEXL(x, mu=0.4089915, sigma=2.710467), add=TRUE,

```

```

      col="tomato", lwd=2)
# Empirical cdf and estimated ecdf
plot(ecdf(x))
curve(pEXL(x, mu=0.4089915, sigma=2.710467), add=TRUE,
      col="tomato", lwd=2)
# QQplot
qqplot(x, rEXL(n=30, mu=0.4089915, sigma=2.710467),
       col="tomato")
qqline(x, distribution=function(p) qEXL(p, mu=0.4089915, sigma=2.710467))

# Example 4
# Precipitation in inches
# Taken from Alomair et al. (2024) page 13.

# Manuel

```

---

 ExW

*The Extended Weibull family*


---

## Description

The Extended Weibull family

## Usage

```
ExW(mu.link = "log", sigma.link = "log", nu.link = "log")
```

## Arguments

mu.link defines the mu.link, with "log" link as the default for the mu parameter.  
 sigma.link defines the sigma.link, with "log" link as the default for the sigma.  
 nu.link defines the nu.link, with "log" link as the default for the nu parameter.

## Details

The Extended Weibull distribution with parameters mu, sigma and nu has density given by

$$f(x) = \frac{\mu\sigma\nu x^{\sigma-1} \exp(-\mu x^\sigma)}{[1-(1-\nu)\exp(-\mu x^\sigma)]^2},$$

for  $x > 0$ .

## Value

Returns a `gamlss.family` object which can be used to fit a ExW distribution in the `gamlss()` function.

## Author(s)

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

## References

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Zhang, T., & Xie, M. (2007). Failure data analysis with extended Weibull distribution. *Communications in Statistics—Simulation and Computation*, 36(3), 579-592.

## See Also

[dExW](#)

## Examples

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rExW(n=200, mu=0.3, sigma=2, nu=0.05)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='ExW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))

# Example 2
# Generating random values under some model
n <- 500
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(-2 + 3 * x1)
sigma <- exp(1.3 - 2 * x2)
nu <- 0.05
x <- rExW(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, family=ExW,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))
```

ExWALD

*The Ex-Wald family***Description**

The function `ExWALD()` defines the Ex-wALD distribution, three-parameter continuous distribution for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

**Usage**

```
ExWALD(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma` parameter.  
`nu.link` defines the `nu.link`, with "log" link as the default for the `nu` parameter.

**Details**

The Ex-Wald distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(x|\mu, \sigma, \nu) = \frac{1}{\nu} \exp\left(\frac{-x}{\nu} + \sigma(\mu - k)\right) F_W(x|k, \sigma) \text{ for } k \geq 0$$

$$f(x|\mu, \sigma, \nu) = \frac{1}{\nu} \exp\left(\frac{-(\sigma-\mu)^2}{2x}\right) Re\left(w(k' \sqrt{x/2} + \frac{\sigma i}{\sqrt{2x}})\right) \text{ for } k < 0$$

where  $k = \sqrt{\mu^2 - \frac{2}{\nu}}$ ,  $k' = \sqrt{\frac{2}{\nu} - \mu^2}$  and  $F_W$  corresponds to the cumulative function of the Wald distribution.

More details about those expressions can be found on page 680 from Heathcote (2004).

**Value**

Returns a `gamlss.family` object which can be used to fit a Ex-WALD distribution in the `gamlss()` function.

**Author(s)**

Freddy Hernandez, <fhernanb@unal.edu.co>

**References**

- Schwarz, W. (2001). The ex-Wald distribution as a descriptive model of response times. *Behavior Research Methods, Instruments, & Computers*, 33, 457-469.
- Heathcote, A. (2004). Fitting Wald and ex-Wald distributions to response time data: An example using functions for the S-PLUS package. *Behavior Research Methods, Instruments, & Computers*, 36, 678-694.



**Description**

The function `FWE()` defines the Flexible Weibull distribution, a two parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

**Usage**

```
FWE(mu.link = "log", sigma.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.

**Details**

The Flexible Weibull extension with parameters `mu` and `sigma` has density given by

$$f(x) = (\mu + \sigma/x^2) \exp(\mu x - \sigma/x) \exp(-\exp(\mu x - \sigma/x))$$

for  $x > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a FWE distribution in the `gamlss()` function.

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
y <- rFWE(n=100, mu=0.75, sigma=1.3)

# Fitting the model
require(gamlss)
mod <- gamlss(y~1, sigma.fo=~1, family='FWE',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))

# Example 2
# Generating random values under some model
n <- 200
```

```

x1 <- runif(n)
x2 <- runif(n)
mu <- exp(1.21 - 3 * x1)
sigma <- exp(1.26 - 2 * x2)
x <- rFWE(n=n, mu, sigma)

mod <- gamlss(x~x1, sigma.fo=~x2, family=FWE,
             control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")

```

GammaW

*The Gamma Weibull family***Description**

The Gamma Weibull family

**Usage**

```
GammaW(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.  
`nu.link` defines the `nu.link`, with "log" link as the default for the `nu` parameter.

**Details**

The Gamma Weibull distribution with parameters `mu`, `sigma` and `nu` has density given by

$$f(x) = \frac{\sigma \mu^\nu}{\Gamma(\nu)} x^{\nu\sigma-1} \exp(-\mu x^\sigma),$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a GammaW distribution in the `gamlss()` function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

## References

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Stacy, E. W. (1962). A generalization of the gamma distribution. *The Annals of mathematical statistics*, 1187-1192.

## See Also

[dGammaW](#)

## Examples

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rGammaW(n=100, mu = 0.5, sigma = 2, nu=1)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='GammaW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n)
x2 <- runif(n)
mu <- exp(1.2 - 1.6 * x1)
sigma <- exp(1.1 - 1 * x2)
nu <- 1
y <- rGammaW(n=n, mu, sigma, nu)

mod <- gamlss(y~x1, mu.fo=~x1, sigma.fo=~x2, nu.fo=~1, family=GammaW,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))
```

GGD

*The Generalized Gompertz family***Description**

The Generalized Gompertz family

**Usage**

```
GGD(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.  
`nu.link` defines the `nu.link`, with "log" link as the default for the `nu` parameter.

**Details**

The Generalized Gompertz Distribution with parameters `mu`, `sigma` and `nu` has density given by

$$f(x) = \nu\mu \exp\left(-\frac{\mu}{\sigma}(\exp(\sigma x - 1))\right) \left(1 - \exp\left(-\frac{\mu}{\sigma}(\exp(\sigma x - 1))\right)\right)^{(\nu-1)},$$

for  $x \geq 0$ ,  $\mu > 0$ ,  $\sigma \geq 0$  and  $\nu > 0$

**Value**

Returns a `gamlss.family` object which can be used to fit a GGD distribution in the `gamlss()` function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

# El-Gohary, A., Alshamrani, A., & Al-Otaibi, A. N. (2013). The generalized Gompertz distribution. *Applied mathematical modelling*, 37(1-2), 13-24.

**See Also**

[dGGD](#)

**Examples**

```

#Example 1
# Generating some random values with
# known mu, sigma, nu and tau
y <- rGGD(n=1000, mu=1, sigma=0.3, nu=1.5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='GGD',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(0.5 - x1)
sigma <- exp(-1 - x2)
nu <- 1.5
x <- rGGD(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, family=GGD,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))

```

**Description**

The Generalized Inverse Weibull family

**Usage**

```
GIW(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.

sigma.link defines the sigma.link, with "log" link as the default for the sigma.  
 nu.link defines the nu.link, with "log" link as the default for the nu parameter.

### Details

The Generalized Inverse Weibull distribution with parameters mu, sigma and nu has density given by

$$f(x) = \nu \sigma \mu^\sigma x^{-(\sigma+1)} \exp\left\{-\nu \left(\frac{\mu}{x}\right)^\sigma\right\},$$

for  $x > 0$ .

### Value

Returns a `gamlss.family` object which can be used to fit a GIW distribution in the `gamlss()` function.

### Author(s)

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

### References

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

De Gusmao, F. R., Ortega, E. M., & Cordeiro, G. M. (2011). The generalized inverse Weibull distribution. *Statistical Papers*, 52, 591-619.

### See Also

[dGIW](#)

### Examples

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rGIW(n=200, mu=3, sigma=5, nu=0.5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='GIW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))

# Example 2
# Generating random values under some model
```

```

n <- 500
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(-1.02 + 3 * x1)
sigma <- exp(1.69 - 2 * x2)
nu <- 0.5
x <- rGIW(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, family=GIW,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))

```

**Description**

The Generalized Lindley Type II (GL2) family for fitting positive continuous lifetime data within the GAMLSS framework.

**Usage**

```
GL2(mu.link = "log", sigma.link = "log")
```

**Arguments**

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the <code>mu</code> parameter ( $\mu > 0$ ).
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the <code>sigma</code> parameter ( $\sigma > 0$ ).

**Details**

The Generalized Lindley Type II distribution with parameters  $\mu$  and  $\sigma$  has probability density function

$$f(x|\mu, \sigma) = \frac{\mu^2}{\mu+1} \left( 1 + \frac{\mu^{\sigma-2} x^{\sigma-1}}{\Gamma(\sigma)} \right) e^{-\mu x},$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ .

The distribution is a two-parameter extension of the classical Lindley distribution and belongs to the class of finite mixtures involving exponential and gamma components. It provides additional flexibility for modeling positively skewed lifetime data.

The original parameters of the distribution are denoted by  $\theta$  and  $\alpha$ . In the GAMLSS implementation, they are re-parameterized as

$$\mu = \theta$$

and

$$\sigma = \alpha.$$

The  $r$ -th raw moment is given by

$$E(X^r) = \frac{1}{\mu^r(\mu+1)} \left[ \mu\Gamma(r+1) + \frac{\Gamma(r+\sigma)}{\Gamma(\sigma)} \right].$$

In particular, the mean is

$$E(X) = \frac{\mu+\sigma}{\mu(\mu+1)}.$$

The GL2 distribution has been proposed for modeling lifetime and survival data and has shown greater flexibility than the classical Lindley and Exponential distributions in several applications.

### Value

Returns a `gamlss.family` object which can be used to fit a GL2 distribution in the `gamlss()` function.

### Author(s)

Sofia Cadavid Rueda, <[socadavidr@unal.edu.co](mailto:socadavidr@unal.edu.co)>

### References

Ekhosuehi, N., Opone, F., & Odobaire, F. (2018). A New Generalized Two Parameter Lindley Distribution. *Journal of the Nigerian Statistical Association*, 30, 547-566.

### See Also

[dGL2](#)

### Examples

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(1234)
y <- rGL2(n=500, mu=0.75, sigma=1.3)

# Fitting the model
require(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=GL2)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with Y as GL2
gendat <- function(n) {
  x1 <- runif(n)
```

```

x2 <- runif(n)
mu <- exp(1.45 - 3 * x1) # Approx 0.95
sigma <- exp(2 - 1.5 * x2) # Approx 3.50
y <- rGL2(n=n, mu=mu, sigma=sigma)
data.frame(y=y, x1=x1, x2=x2)
}
set.seed(1234)
dat <- gendat(n=1000)

mod2 <- gamlss(y~x1, sigma.fo=~x2,
              family=GL2, data=dat,
              control=gamlss.control(n.cyc=50, trace=FALSE))

summary(mod2)

# Example 3
# Remission times (in months) of 128 bladder cancer patients
# Taken from Lee and Wang (2003)
# Ekhosuehi et al. (2018) Table 4

y <- c(0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63,
       0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40,
       2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50,
       2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51,
       2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81,
       2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64,
       3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69,
       4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69,
       4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75,
       4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33,
       5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62,
       7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93,
       11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13,
       1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31,
       4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76,
       12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69)

require(gamlss)
mod3 <- gamlss(y ~ 1, sigma.fo = ~1, family = GL2)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod3, what="mu"))
exp(coef(mod3, what="sigma"))

# Comparing the empirical histogram with the estimated density
hist(y, breaks=15, freq=FALSE,
     xlab="y", col="lightblue", border="white")
curve(dGL2(x, mu=0.1247079, sigma=1.189219),
      add=TRUE, col="red", lwd=2)

```

**Description**

The Generalized modified Weibull distribution

**Usage**

```
GMW(mu.link = "log", sigma.link = "log", nu.link = "sqrt", tau.link = "sqrt")
```

**Arguments**

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the <code>mu</code> parameter.
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the <code>sigma</code> .
<code>nu.link</code>	defines the <code>nu.link</code> , with "sqrt" link as the default for the <code>nu</code> parameter.
<code>tau.link</code>	defines the <code>tau.link</code> , with "sqrt" link as the default for the <code>tau</code> parameter.

**Details**

The Generalized modified Weibull distribution with parameters `mu`, `sigma`, `nu` and `tau` has density given by

$$f(x) = \mu\sigma x^{\nu-1}(\nu + \tau x) \exp(\tau x - \mu x^{\nu} e^{\tau x}) [1 - \exp(-\mu x^{\nu} e^{\tau x})]^{\sigma-1},$$

for  $x > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a GMW distribution in the `gamlss()` function.

**See Also**

[dGMW](#)

**Examples**

```
# Example 1
# Generating some random values with
# known mu, sigma, nu and tau
y <- rGMW(n=100, mu=2, sigma=0.5, nu=2, tau=1.5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, tau.fo=~ 1, family='GMW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))
```

```

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
(coef(mod, what='nu'))^2
(coef(mod, what='tau'))^2

# Example 2
# Generating random values under some model
## Not run:
n <- 1000
x1 <- runif(n)
x2 <- runif(n)
mu <- exp(2 + -3 * x1)
sigma <- exp(3 - 2 * x2)
nu <- 2
tau <- 1.5
x <- rGMW(n=n, mu, sigma, nu, tau)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, tau.fo=~ 1, family="GMW",
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
coef(mod, what="nu")^2
coef(mod, what="tau")^2

## End(Not run)

```

---

initValuesOW

*Initial values and search region for Odd Weibull distribution*


---

### Description

This function can be used so as to get suggestions about initial values and the search region for parameter estimation in OW distribution.

### Usage

```

initValuesOW(
  formula,
  data = NULL,
  local_reg = loess.options(),
  interpolation = interp.options(),
  ...
)

```

**Arguments**

formula	an object of class <code>formula</code> with the response on the left of an operator $\sim$ . The right side must be 1.
data	an optional data frame containing the response variables. If data is not specified, the variables are taken from the environment from which <code>initValuesOW</code> is called.
local_reg	a list of control parameters for LOESS. See <code>loess.options</code> .
interpolation	a list of control parameters for interpolation function. See <code>interp.options</code> .
...	further arguments passed to <code>TTTE_Analytical</code> .

**Details**

This function performs a non-parametric estimation of the empirical total time on test (TTT) plot. Then, this estimated curve can be used so as to get suggestions about initial values and the search region for parameters based on hazard shape associated to the shape of empirical TTT plot.

**Value**

Returns an object of class `c("initValOW", "HazardShape")` containing:

- `sigma.start` value for *sigma* parameter of OW distribution.
- `nu.start` value for *nu* parameter of OW distribution.
- `sigma.valid` search region for *sigma* parameter of OW distribution.
- `nu.valid` search region for *nu* parameter of OW distribution.
- `TTTplot` Total Time on Test transform computed from the data.
- `hazard_type` shape of the hazard function determined from the TTT plot.

**Author(s)**

Jaime Mosquera Gutiérrez <jmosquerag@unal.edu.co>

**Examples**

```
# Example 1
# Bathtub hazard and its corresponding TTT plot
y1 <- rOW(n = 1000, mu = 0.1, sigma = 7, nu = 0.08)
my_initial_guess1 <- initValuesOW(formula=y1~1)
summary(my_initial_guess1)
plot(my_initial_guess1, par_plot=list(mar=c(3.7,3.7,1,2.5),
                                     mgp=c(2.5,1,0)))

curve(hOW(x, mu = 0.022, sigma = 8, nu = 0.01), from = 0,
      to = 80, ylim = c(0, 0.04), col = "red",
      ylab = "Hazard function", las = 1)

# Example 2
# Bathtub hazard and its corresponding TTT plot with right censored data
```

```

y2 <- rOW(n = 1000, mu = 0.1, sigma = 7, nu = 0.08)
status <- c(rep(1, 980), rep(0, 20))
my_initial_guess2 <- initialValuesOW(formula=Surv(y2, status)~1)
summary(my_initial_guess2)
plot(my_initial_guess2, par_plot=list(mar=c(3.7,3.7,1,2.5),
                                     mgp=c(2.5,1,0)))

curve(hOW(x, mu = 0.022, sigma = 8, nu = 0.01), from = 0,
      to = 80, ylim = c(0, 0.04), col = "red",
      ylab = "Hazard function", las = 1)

```

---

 IW

*The Inverse Weibull family*


---

## Description

The Inverse Weibull distribution

## Usage

```
IW(mu.link = "log", sigma.link = "log")
```

## Arguments

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the <code>mu</code> parameter.
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the <code>sigma</code> .

## Details

The Inverse Weibull distribution with parameters `mu`, `sigma` has density given by

$$f(x) = \mu\sigma x^{-\sigma-1} \exp(\mu x^{-\sigma})$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$

## Value

Returns a `gamlss.family` object which can be used to fit a IW distribution in the `gamlss()` function.

## Author(s)

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

## References

- Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.
- Drapella, A. (1993). The complementary Weibull distribution: unknown or just forgotten?. *Quality and reliability engineering international*, 9(4), 383-385.

**See Also**[dIW](#)**Examples**

```

# Example 1
# Generating some random values with
# known mu and sigma
y <- rIW(n=100, mu=5, sigma=2.5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, mu.fo=~1, sigma.fo=~1, family='IW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- rpois(n, lambda=2)
x2 <- runif(n)
mu <- exp(2 + -1 * x1)
sigma <- exp(2 - 2 * x2)
x <- rIW(n=n, mu, sigma)

mod <- gamlss(x~x1, mu.fo=~1, sigma.fo=~x2, family=IW,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")

```

KumIW

*The Kumaraswamy Inverse Weibull family***Description**

The Kumaraswamy Inverse Weibull family

**Usage**

```
KumIW(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the mu parameter.
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the sigma.
<code>nu.link</code>	defines the <code>nu.link</code> , with "log" link as the default for the nu parameter.

**Details**

The Kumaraswamy Inverse Weibull Distribution with parameters `mu`, `sigma` and `nu` has density given by

$$f(x) = \mu\sigma\nu x^{-\mu-1} \exp -\sigma x^{-\mu} (1 - \exp -\sigma x^{-\mu})^{\nu-1},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a KumIW distribution in the `gamlss()` function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

- Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.
- Shahbaz, M. Q., Shahbaz, S., & Butt, N. S. (2012). The Kumaraswamy Inverse Weibull Distribution. *Pakistan journal of statistics and operation research*, 479-489.

**See Also**

[dKumIW](#)

**Examples**

```
# Example 1
# Generating some random values with
# known mu, sigma, nu and tau
y <- rKumIW(n=100, mu = 1.5, sigma= 1.5, nu = 5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family="KumIW",
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what="mu"))
exp(coef(mod, what="sigma"))
```

```

exp(coef(mod, what="nu"))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(1 - x1)
sigma <- exp(1 - x2)
nu <- 5
x <- rKumIW(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, family=KumIW,
             control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))

```

---

 LIN

*The Lindley family*


---

### Description

The function LIN() defines the Lindley distribution with only one parameter for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

### Usage

```
LIN(mu.link = "log")
```

### Arguments

mu.link defines the mu.link, with "log" link as the default for the mu parameter.

### Details

The Lindley with parameter mu has density given by

$$f(x) = \frac{\mu^2}{\mu+1} (1+x) \exp(-\mu x),$$

for  $x > 0$  and  $\mu > 0$ .

### Value

Returns a gamlss.family object which can be used to fit a LIN distribution in the gamlss() function.

### Author(s)

Freddy Hernandez <fhernanb@unal.edu.co>

## References

Lindley, D. V. (1958). Fiducial distributions and Bayes' theorem. *Journal of the Royal Statistical Society. Series B (Methodological)*, 102-107.

## Examples

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rLIN(n=200, mu=2)

# Fitting the model
require(gamlss)
mod <- gamlss(y ~ 1, family="LIN")

# Extracting the fitted values for mu
# using the inverse link function
exp(coef(mod, what='mu'))

# Example 2
# Generating random values under some model
n <- 100
x1 <- runif(n=n)
x2 <- runif(n=n)
eta <- 1 + 3 * x1 - 2 * x2
mu <- exp(eta)
y <- rLIN(n=n, mu=mu)

mod <- gamlss(y ~ x1 + x2, family=LIN)

coef(mod, what='mu')
```

---

logLik\_GL2

*logLik\_GL2*


---

## Description

Auxiliary function to compute the log-likelihood of the GL2 distribution.

## Usage

```
logLik_GL2(param = c(0, 0), x)
```

## Arguments

param	Numeric vector containing the values of the parameters
x	Numeric vector containing the observations.

**Examples**

```
y <- rGL2(n = 100, mu = 3, sigma = 1.2)
logLik_GL2(param = c(0, 0), x = y)
```

LW

*The Log-Weibull family***Description**

The Log-Weibull distribution

**Usage**

```
LW(mu.link = "identity", sigma.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.

**Details**

The Log-Weibull Distribution with parameters `mu` and `sigma` has density given by

$$f(y) = (1/\sigma)e^{((y-\mu)/\sigma)} \exp\{-e^{((y-\mu)/\sigma)}\},$$

for  $-\infty < y < \infty$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a LW distribution in the `gamlss()` function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Gumbel, E. J. (1958). *Statistics of extremes*. Columbia university press.

**See Also**

[dLW](#)

**Examples**

```

# Example 1
# Generating some random values with
# known mu and sigma
y <- rLW(n=100, mu=0, sigma=1.5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, family= 'LW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu and sigma
# using the inverse link function
coef(mod, 'mu')
exp(coef(mod, 'sigma'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- 1.5 - 3 * x1
sigma <- exp(1.4 - 2 * x2)
x <- rLW(n=n, mu, sigma)

mod <- gamlss(x~x1, sigma.fo=~x2, family=LW,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")

```

---

mice

*Mice mortality data*


---

**Description**

The ages at death in weeks for male mice exposed to 240r of gamma radiation.

**Usage**

```
data(mice)
```

**Format**

A vector with 208 data points.

**Examples**

```
data(mice)
hist(mice, main="", xlab="Time (weeks)", freq=FALSE)
lines(density(mice), col="blue", lwd=2)
```

MOEIW

*The Marshall-Olkin Extended Inverse Weibull family***Description**

The Marshall-Olkin Extended Inverse Weibull family

**Usage**

```
MOEIW(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.  
`nu.link` defines the `nu.link`, with "log" link as the default for the `nu` parameter.

**Details**

The Marshall-Olkin Extended Inverse Weibull distribution with parameters `mu`, `sigma` and `nu` has density given by

$$f(x) = \frac{\mu\sigma\nu x^{-(\sigma+1)} \exp\{-\mu x^{-\sigma}\}}{\{\nu - (\nu-1)\exp\{-\mu x^{-\sigma}\}\}^2},$$

for  $x > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a MOEIW distribution in the `gamlss()` function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Okasha, H. M., El-Baz, A. H., Tarabia, A. M. K., & Basheer, A. M. (2017). Extended inverse Weibull distribution with reliability application. *Journal of the Egyptian Mathematical Society*, 25(3), 343-349.

**See Also**

[dMOEIW](#)

**Examples**

```

# Example 1
# Generating some random values with
# known mu, sigma and nu
set.seed(123456)
y <- rMOEIW(n=100, mu=0.6, sigma=1.7, nu=0.3)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family="MOEIW",
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what="mu"))
exp(coef(mod, what="sigma"))
exp(coef(mod, what="nu"))

# Example 2
# Generating random values under some model

# A function to simulate a data set with  $Y \sim \text{MOEIW}$ 
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu   <- exp(-2.02 + 3 * x1) # 0.60 approximately
  sigma <- exp(2.23 - 2 * x2) # 3.42 approximately
  nu   <- 2
  y <- rMOEIW(n=n, mu=mu, sigma=sigma, nu=nu)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(123)
dat <- gendat(n=100)

mod <- gamlss(y~x1, sigma.fo=~x2, nu.fo=~1,
              family=MOEIW, data=dat,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))

```

**Description**

The Marshall-Olkin Extended Weibull family

**Usage**

```
MOEW(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

mu.link	defines the mu.link, with "log" link as the default for the mu parameter.
sigma.link	defines the sigma.link, with "log" link as the default for the sigma.
nu.link	defines the nu.link, with "log" link as the default for the nu parameter.

**Details**

The Marshall-Olkin Extended Weibull distribution with parameters mu, sigma and nu has density given by

$$f(x) = \frac{\mu\sigma\nu(\nu x)^{\sigma-1} \exp\{-(\nu x)^\sigma\}}{\{1-(1-\mu)\exp\{-(\nu x)^\sigma\}\}^2},$$

for  $x > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a MOEW distribution in the `gamlss()` function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Ghitany, M. E., Al-Hussaini, E. K., & Al-Jarallah, R. A. (2005). Marshall–Olkin extended Weibull distribution and its application to censored data. *Journal of Applied Statistics*, 32(10), 1025-1034.

**See Also**

[dMOEW](#)

**Examples**

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rMOEW(n=400, mu=0.5, sigma=0.7, nu=1)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='MOEW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))
```

```

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))

# Example 2
# Generating random values under some model
n <- 500
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(-1.20 + 3 * x1)
sigma <- exp(0.84 - 2 * x2)
nu <- 1
x <- rMOEW(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, family=MOEW,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))

```

MOK

*The Marshall-Olkin Kappa family***Description**

The Marshall-Olkin Kappa family

**Usage**

```
MOK(mu.link = "log", sigma.link = "log", nu.link = "log", tau.link = "log")
```

**Arguments**

mu.link	defines the mu.link, with "log" link as the default for the mu parameter.
sigma.link	defines the sigma.link, with "log" link as the default for the sigma.
nu.link	defines the nu.link, with "log" link as the default for the nu parameter.
tau.link	defines the tau.link, with "log" link as the default for the tau parameter.

**Details**

The Marshall-Olkin Kappa distribution with parameters mu, sigma, nu and tau has density given by

$$f(x) = \frac{\tau \frac{\mu\nu}{\sigma} \left(\frac{x}{\sigma}\right)^{\nu-1} \left(\mu + \left(\frac{x}{\sigma}\right)^{\mu\nu}\right)^{-\frac{\mu+1}{\mu}}}{\left(\tau + (1-\tau) \left(\frac{\left(\frac{x}{\sigma}\right)^{\mu\nu}}{\mu + \left(\frac{x}{\sigma}\right)^{\mu\nu}}\right)^{\frac{1}{\mu}}\right)^2}$$

for  $x > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a MOK distribution in the `gamlss()` function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

Javed, M., Nawaz, T., & Irfan, M. (2019). The Marshall-Olkin kappa distribution: properties and applications. *Journal of King Saud University-Science*, 31(4), 684-691.

**See Also**

[dMOK](#)

**Examples**

```
# Example 1
# Generating some random values with
# known mu, sigma, nu and tau
y <- rMOK(n=100, mu = 1, sigma = 3.5, nu = 3, tau = 2)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, tau.fo=~1, family=MOK,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma, nu and tau
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))
exp(coef(mod, what='tau'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(0.5 + x1)
sigma <- exp(0.8 + x2)
nu <- 1
tau <- 0.5
x <- rMOK(n=n, mu, sigma, nu, tau)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, tau.fo=~1, family=MOK,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
```

```
coef(mod, what="sigma")
exp(coef(mod, what="nu"))
exp(coef(mod, what="tau"))
```

---

 MW

*The Modified Weibull family*


---

## Description

#' The Modified Weibull distribution

## Usage

```
MW(mu.link = "log", sigma.link = "log", nu.link = "log")
```

## Arguments

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.  
`nu.link` defines the `nu.link`, with "log" link as the default for the `nu` parameter.

## Details

The Modified Weibull distribution with parameters `mu`, `sigma` and `nu` has density given by

$$f(x) = \mu(\sigma + \nu x)x^{\sigma-1} \exp(\nu x) \exp(-\mu x^\sigma \exp(\nu x)),$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma \geq 0$  and  $\nu \geq 0$ .

## Value

Returns a `gamlss.family` object which can be used to fit a MW distribution in the `gamlss()` function.

## Author(s)

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

## References

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Lai, C. D., Xie, M., & Murthy, D. N. P. (2003). A modified Weibull distribution. *IEEE Transactions on reliability*, 52(1), 33-37.

## See Also

[dMW](#)

**Examples**

```

# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rMW(n=100, mu = 2, sigma = 1.5, nu = 0.2)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family= 'MW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- rpois(n, lambda=2)
x2 <- runif(n)
mu <- exp(3 - 1 * x1)
sigma <- exp(2 - 2 * x2)
nu <- 0.2
x <- rMW(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, mu.fo=~x1, sigma.fo=~x2, nu.fo=~1, family=MW,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
coef(mod, what='nu')

```

---

myOW\_region

*Customized region search for odd Weibull distribution*


---

**Description**

This function can be used to modify OW `gamlss.family` object in order to set a customized region search for `gamlss()` function.

**Usage**

```
myOW_region(family = OW, valid.values = "auto", initVal)
```

**Arguments**

family	The <i>OW</i> family. This arguments allows the user to modify input arguments of the family, like the link functions.
valid.values	a list of character elements specifying the region for sigma and/or nu. See <b>Details</b> and <b>Examples</b> section to learn about its use.
initVal	An initValOW object generated with <code>initValuesOW</code> function.

**Details**

This function was created to help users to fit OW distribution easily bounding the parametric space for sigma and nu.

The `valid.values` must be defined as a list of characters containing a call of the `all` function.

**Value**

Returns a `gamlss.family` object which can be used to fit an OW distribution in the `gamlss()` function.

**Author(s)**

Jaime Mosquera Gutiérrez <jmosquerag@unal.edu.co>

**Examples**

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rOW(n=200, mu=0.2, sigma=4, nu=0.05)

# Custom search region
myvalues <- list(sigma="all(sigma > 1)",
                 nu="all(nu < 1) & all(nu < 1)")

my_initial_guess <- initValuesOW(formula=y~1)
summary(my_initial_guess)

# OW family modified with 'myOW_region'
require(gamlss)
myOW <- myOW_region(valid.values=myvalues, initVal=my_initial_guess)
mod1 <- gamlss(y~1, sigma.fo=~1, nu.fo=~1,
              sigma.start=param.startOW('sigma', my_initial_guess),
              nu.start=param.startOW('nu', my_initial_guess),
              control=gamlss.control(n.cyc=300, trace=FALSE),
              family=myOW)

exp(coef(mod1, what='mu'))
exp(coef(mod1, what='sigma'))
exp(coef(mod1, what='nu'))

# Example 2
```

```
# Same example using another link function and using 'myOW_region'
# in the argument 'family'
mod2 <- gamlss(y~1, sigma.fo=~1, nu.fo=~1,
              sigma.start=2, nu.start=0.1,
              control=gamlss.control(n.cyc=300, trace=FALSE),
              family=myOW_region(family=OW(sigma.link='identity'),
                                valid.values=myvalues,
                                initVal=my_initial_guess))

exp(coef(mod2, what='mu'))
coef(mod2, what='sigma')
exp(coef(mod2, what='nu'))
```

NEE

*New Exponentiated Exponential family***Description**

The function `NEE()` defines the New Exponentiated Exponential distribution, a two parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

**Usage**

```
NEE(mu.link = "log", sigma.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "logit" link as the default for the `sigma`.

**Details**

The New Exponentiated Exponential distribution with parameters `mu` and `sigma` has density given by

$$f(x|\mu, \sigma) = \log(2^\sigma) \mu \exp(-\mu x) (1 - \exp(-\mu x))^{\sigma-1} 2^{(1-\exp(-\mu x))^\sigma},$$

for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ .

Note: In this implementation we changed the original parameters  $\theta$  for  $\mu$  and  $\alpha$  for  $\sigma$ , we did it to implement this distribution within `gamlss` framework.

**Value**

Returns a `gamlss.family` object which can be used to fit a NEE distribution in the `gamlss()` function.

**References**

Hassan, Anwar, I. H. Dar, and M. A. Lone. "A New Class of Probability Distributions With An Application to Engineering Data." *Pakistan Journal of Statistics and Operation Research* 20.2 (2024): 217-231.

**See Also**[dNEE](#)**Examples**

```

# Example 1
# Generating some random values with
# known mu and sigma
y <- rNEE(n=500, mu=2.5, sigma=3.5)

# Fitting the model
require(gamlss)

mod1 <- gamlss(y~1, sigma.fo=~1, family=NEE,
               control=gamlss.control(n.cyc=5000, trace=TRUE))

# Extracting the fitted values for mu, sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values under some model
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(-0.2 + 1.5 * x1)
  sigma <- exp(1 - 0.7 * x2)
  y <- rNEE(n=n, mu, sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

set.seed(123)
datos <- gendat(n=500)

mod2 <- gamlss(y~x1, sigma.fo=~x2, family=NEE, data=datos,
               control=gamlss.control(n.cyc=5000, trace=TRUE))

summary(mod2)

# Example 3 -----
# Obtained from Hassan (2024) page 226
# The data set consists of 63 observations of the gauge lengths of 10mm.

y <- c(1.901, 2.132, 2.203, 2.228, 2.257, 2.350, 2.361, 2.396, 2.397,
       2.445, 2.454, 2.474, 2.518, 2.522, 2.525, 2.532, 2.575, 2.614,
       2.616, 2.618, 2.624, 2.659, 2.675, 2.738, 2.740, 2.856, 2.917,
       2.928, 2.937, 2.937, 2.977, 2.996, 3.030, 3.125, 3.139, 3.145,
       3.220, 3.223, 3.235, 3.243, 3.264, 3.272, 3.294, 3.332, 3.346,
       3.377, 3.408, 3.435, 3.493, 3.501, 3.537, 3.554, 3.562, 3.628,
       3.852, 3.871, 3.886, 3.971, 4.024, 4.027, 4.225, 4.395, 5.020)

```

```

mod3 <- gamlss(y~1, family=NEE)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod3, what="mu"))
exp(coef(mod3, what="sigma"))

# Hist and estimated pdf
hist(y, freq=FALSE, ylim=c(0, 0.7))
curve(dNEE(x, mu=2.076862, sigma=255.2289),
      add=TRUE, col="tomato", lwd=2)

# Empirical cdf and estimated ecdf
plot(ecdf(y))
curve(pNEE(x, mu=2.076862, sigma=255.2289),
      add=TRUE, col="tomato", lwd=2)
# QQplot
qqplot(y, rNEE(n=length(y), mu=2.076862, sigma=255.2289), col="tomato")
qqline(y, distribution=function(p) qNEE(p, mu=2.076862, sigma=255.2289))

# Example 4 -----
# Obtained from Hassan (2024) page 226
# The dataset was reported by Bader and Priest (1982) on failure
# stresses (in GPa) of 65 single carbon fibers of lengths 50 mm

y <- c(0.564, 0.729, 0.802, 0.95, 1.053, 1.111, 1.115, 1.194, 1.208,
      1.216, 1.247, 1.256, 1.271, 1.277, 1.305, 1.313, 1.348,
      1.39, 1.429, 1.474, 1.49, 1.503, 1.52, 1.522, 1.524, 1.551,
      1.551, 1.609, 1.632, 1.632, 1.676, 1.684, 1.685, 1.728, 1.74,
      1.761, 1.764, 1.785, 1.804, 1.816, 1.824, 1.836, 1.879, 1.883,
      1.892, 1.898, 1.934, 1.947, 1.976, 2.02, 2.023, 2.05, 2.059,
      2.068, 2.071, 2.098, 2.13, 2.204, 2.317, 2.334, 2.34, 2.346,
      2.378, 2.483, 2.269)

mod4 <- gamlss(y~1, family=NEE)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod4, what="mu"))
exp(coef(mod4, what="sigma"))

hist(y, freq=FALSE)
curve(dNEE(x, mu=2.400515, sigma=25.15236),
      add=TRUE, col="tomato", lwd=2)

# Empirical cdf and estimated ecdf
plot(ecdf(y))
curve(pNEE(x, mu=2.400515, sigma=25.15236),
      add=TRUE, col="tomato", lwd=2)
# QQplot
qqplot(y, rNEE(n=length(y), mu=2.400515, sigma=25.15236), col="tomato")
qqline(y, distribution=function(p) qNEE(p, mu=2.400515, sigma=25.15236))

```

```

# Example 5 -----
# 69 Observations of the gauge lengths of 20m.
y <- c(1.312,1.314,1.479,1.552,1.700,1.803,1.861,1.865,1.944,1.958,1.966,1.997,
      2.006,2.021,2.027,2.055, 2.063,2.098,2.140,2.179,2.224,2.240,2.253,2.270,
      2.272,2.274,2.301,2.301,2.359,2.382,2.382,2.426, 2.434,2.435,2.478,2.490,
      2.511,2.514,2.535,2.554,2.566,2.570,2.586,2.629,2.633,2.642,2.648,2.684,
      2.697,2.726,2.770,2.773,2.800,2.809,2.818,2.821,2.848,2.880,2.954,3.012,
      3.067,3.084,3.090,3.096, 3.128,3.233,3.433,3.585,3.585)

mod5 <- gamlss(y~1, sigma.fo=~1, family = NEE)

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod5, what="mu"))
exp(coef(mod5, what="sigma"))

hist(y, freq=FALSE)
curve(dNEE(x, mu=2.197771, sigma=100.8888), add=TRUE,
      col="tomato", lwd=2)
# Empirical cdf and estimated ecdf
plot(ecdf(y))
curve(pNEE(x, mu=2.197771, sigma=100.8888), add=TRUE,
      col="tomato", lwd=2)
# QQplot
qqplot(y, rNEE(n=length(y), mu=2.197771, sigma=100.8888), col="tomato")
qqline(y, distribution=function(p) qNEE(p, mu=2.197771, sigma=100.8888))

```

**Description**

The function `OW()` defines the Odd Weibull distribution, a three parameter distribution, for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

**Usage**

```
OW(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the <code>mu</code> parameter.
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the <code>sigma</code> .
<code>nu.link</code>	defines the <code>nu.link</code> , with "log" link as the default for the <code>nu</code> .

**Details**

The odd Weibull with parameters  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(t) = \left(\frac{\sigma\nu}{t}\right) (\mu t)^\sigma e^{(\mu t)^\sigma} (e^{(\mu t)^\sigma} - 1)^{\nu-1} \left[1 + (e^{(\mu t)^\sigma} - 1)^\nu\right]^{-2}$$

for  $x > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a OW distribution in the `gamlss()` function.

**Author(s)**

Jaime Mosquera Gutiérrez <jmosquerag@unal.edu.co>

**References**

Cooray, K. (2006). Generalization of the Weibull distribution: the odd Weibull family. *Statistical Modelling*, 6(3), 265-277.

**Examples**

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rOW(n=200, mu=0.1, sigma=7, nu = 1.1)

# Fitting the model
require(gamlss)
mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family="OW",
             control=gamlss.control(n.cyc=500, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what="mu"))
exp(coef(mod, what="sigma"))
exp(coef(mod, what="nu"))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n)
x2 <- runif(n)
x3 <- runif(n)
mu <- exp(1.2 + 2 * x1)
sigma <- 2.12 + 3 * x2
nu <- exp(0.2 - x3)
y <- rOW(n=n, mu, sigma, nu)

mod <- gamlss(y~x1, sigma.fo=~x2, nu.fo=~x3,
             family=OW(sigma.link="identity"),
             control=gamlss.control(n.cyc=300, trace=FALSE))
```

```
coef(mod, what="mu")
coef(mod, what="sigma")
coef(mod, what="nu")
```

---

param.startOW	<i>Initial values extraction for Odd Weibull distribution</i>
---------------	---

---

### Description

This function can be used to extract initial values found with empirical time on test transform (TTT) through [initValuesOW](#) function. This is used for parameter estimation in OW distribution.

### Usage

```
param.startOW(param, initValOW)
```

### Arguments

param	a character used to specify the parameter required. It can take the values "sigma" or "nu".
initValOW	an <a href="#">initValOW</a> object generated with <a href="#">initValuesOW</a> function.

### Details

This function just gets initial values computed with [initValuesOW](#) for OW family. It must be called in `sigma.start` and `nu.start` arguments from [gamlss](#) function. This function is useful only if user want to set start values automatically with TTT plot. See example for an illustration.

### Value

A length-one vector numeric value corresponding to the initial value of the parameter specified in `param` extracted from a [initValuesOW](#) object specified in the `initValOW` input argument.

### Author(s)

Jaime Mosquera Gutiérrez <jmosquerag@unal.edu.co>

### Examples

```
# Random data generation (OW distributed)
y <- rOW(n=500, mu=0.05, sigma=0.6, nu=2)

# Initial values with TTT plot
iv <- initValuesOW(formula = y ~ 1)
summary(iv)

# This data is from unimodal hazard
# See TTT estimate from sample
```

```

plot(iv, legend_options=list(pos=1.03))

# See the true hazard
curve(hOW(x, mu=0.05, sigma=0.6, nu=2), to=100, lwd=3, ylab="h(x)")

# Finally, we fit the model
require(gamlss)
con.out <- gamlss.control(n.cyc = 300, trace = FALSE)
con.in <- glim.control(cyc = 300)

(sigma.start <- param.startOW("sigma", iv))
(nu.start <- param.startOW("nu", iv))

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, control=con.out, i.control=con.in,
             family=myOW_region(OW(sigma.link="identity", nu.link="identity"),
                                valid.values="auto", iv),
             sigma.start=sigma.start, nu.start=nu.start)

# Estimates are close to actual values
(mu <- exp(coef(mod, what = "mu")))
(sigma <- coef(mod, what = "sigma"))
(nu <- coef(mod, what = "nu"))

```

---

PL

*The Power Lindley family*


---

## Description

Power Lindley distribution

## Usage

```
PL(mu.link = "log", sigma.link = "log")
```

## Arguments

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.

## Details

The Power Lindley Distribution with parameters `mu` and `sigma` has density given by

$$f(x) = \frac{\mu\sigma^2}{\sigma+1}(1+x^\mu)x^{\mu-1}\exp(-\sigma x^\mu),$$

for  $x > 0$ .

## Value

Returns a `gamlss.family` object which can be used to fit a PL distribution in the `gamlss()` function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Ghitany, M. E., Al-Mutairi, D. K., Balakrishnan, N., & Al-Enezi, L. J. (2013). Power Lindley distribution and associated inference. *Computational Statistics & Data Analysis*, 64, 20-33.

**See Also**

[dPL](#)

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
y <- rPL(n=100, mu=1.5, sigma=0.2)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, family= 'PL',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod, 'mu'))
exp(coef(mod, 'sigma'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(1.2 - 2 * x1)
sigma <- exp(0.8 - 3 * x2)
x <- rPL(n=n, mu, sigma)

mod <- gamlss(x~x1, sigma.fo=~x2, family=PL,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
```

QXGP

*The Quasi XGamma Poisson family***Description**

The Quasi XGamma Poisson family

**Usage**

```
QXGP(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

mu.link defines the mu.link, with "log" link as the default for the mu parameter.  
 sigma.link defines the sigma.link, with "log" link as the default for the sigma.  
 nu.link defines the nu.link, with "log" link as the default for the nu parameter.

**Details**

The Quasi XGamma Poisson distribution with parameters mu, sigma and nu has density given by

$$f(x) = K(\mu, \sigma, \nu) \left( \frac{\sigma^2 x^2}{2} + \mu \right) \exp\left( \frac{\nu \exp(-\sigma x) (1 + \mu + \sigma x + \frac{\sigma^2 x^2}{2})}{1 + \mu} - \sigma x \right),$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$ ,  $\nu > 1$ .

where

$$K(\mu, \sigma, \nu) = \frac{\nu \sigma}{(\exp(\nu) - 1)(1 + \mu)}$$

**Value**

Returns a gamlss.family object which can be used to fit a QXGP distribution in the gamlss() function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Sen, S., Korkmaz, M. Ç., & Yousof, H. M. (2018). The quasi XGamma-Poisson distribution: properties and application. *Istatistik Journal of The Turkish Statistical Association*, 11(3), 65-76.

**See Also**

[dQXGP](#)

**Examples**

```

# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rQXGP(n=200, mu=4, sigma=2, nu=3)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='QXGP',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))

# Example 2
# Generating random values under some model
n <- 2000
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(-2.19 + 3 * x1)
sigma <- exp(1 - 2 * x2)
nu <- 1
x <- rQXGP(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, family=QXGP,
              control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))

```

RW

*The Reflected Weibull family***Description**

Reflected Weibull distribution

**Usage**

RW(mu.link = "log", sigma.link = "log")

**Arguments**

mu.link            defines the mu.link, with "log" link as the default for the mu parameter.  
sigma.link        defines the sigma.link, with "log" link as the default for the sigma.

**Details**

The Reflected Weibull Distribution with parameters  $\mu$  and  $\sigma$  has density given by

$$f(y) = \mu\sigma(-y)^{\sigma-1}e^{-\mu(-y)^\sigma},$$

for  $y < 0$

**Value**

Returns a `gamlss.family` object which can be used to fit a RW distribution in the `gamlss()` function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.

Cohen, A. C. (1973). The reflected Weibull distribution. *Technometrics*, 15(4), 867-873.

**See Also**

[dRW](#)

**Examples**

```
# Example 1
# Generating some random values with
# known mu and sigma
y <- rRW(n=100, mu=1, sigma=1)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, family= 'RW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod, 'mu'))
exp(coef(mod, 'sigma'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n, min=0.4, max=0.6)
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(1.5 - 1.5 * x1)
sigma <- exp(2 - 2 * x2)
x <- rRW(n=n, mu, sigma)
```

```

mod <- gamlss(x~x1, sigma.fo=~x2, family=RW,
             control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")

```

---

summary.initValOW      *Summary of initValOW objects*

---

### Description

This summary method displays initial values and search regions for [OW](#) family.

### Usage

```

## S3 method for class 'initValOW'
summary(object, ...)

```

### Arguments

object	an object of class <code>initVal</code> , generated with <a href="#">initValuesOW</a> .
...	extra arguments

### Value

No return value, it just prints out in the console the initial values and the search regions for *sigma* and *nu* from OW distribution (see [dOW](#)).

### Author(s)

Jaime Mosquera Gutiérrez <jmosquerag@unal.edu.co>

---

SZMW      *The Sarhan and Zaindin's Modified Weibull family*

---

### Description

The Sarhan and Zaindin's Modified Weibull distribution

### Usage

```

SZMW(mu.link = "log", sigma.link = "log", nu.link = "log")

```

**Arguments**

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the mu parameter.
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the sigma.
<code>nu.link</code>	defines the <code>nu.link</code> , with "log" link as the default for the nu parameter.

**Details**

The Sarhan and Zaindin's Modified Weibull distribution with parameters `mu`, `sigma` and `nu` has density given by

$$f(x) = (\mu + \sigma\nu x^{\nu-1}) \exp(-\mu x - \sigma x^{\nu}),$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a SZMW distribution in the `gamlss()` function.

**Author(s)**

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

**References**

- Almalki, S. J., & Nadarajah, S. (2014). Modifications of the Weibull distribution: A review. *Reliability Engineering & System Safety*, 124, 32-55.
- Sarhan, A. M., & Zaindin, M. (2009). Modified Weibull distribution. *APPS. Applied Sciences*, 11, 123-136.

**See Also**

[dSZMW](#)

**Examples**

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rSZMW(n=100, mu = 1, sigma = 1, nu = 1.5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='SZMW',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
```

```

exp(coef(mod, what='nu'))

# Example 2
# Generating random values under some model
n     <- 200
x1    <- runif(n)
x2    <- runif(n)
mu    <- exp(-1.6 * x1)
sigma <- exp(0.9 - 1 * x2)
nu    <- 1.5
x     <- rSZMW(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, mu.fo=~x1, sigma.fo=~x2, nu.fo=~1, family=SZMW,
             control=gamlss.control(n.cyc=50000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
coef(mod, what='nu')

```

---

WALD

*The Wald family*


---

### Description

The function `WALD()` defines the `wALD` distribution, two-parameter continuous distribution for a `gamlss.family` object to be used in GAMLSS fitting using the function `gamlss()`.

### Usage

```
WALD(mu.link = "log", sigma.link = "log")
```

### Arguments

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma` parameter.

### Details

The Wald distribution with parameters  $\mu$  and  $\sigma$  has density given by

$$f(x|\mu, \sigma) = \frac{\sigma}{\sqrt{2\pi x^3}} \exp\left[-\frac{(\sigma - \mu x)^2}{2x}\right], x > 0$$

### Value

Returns a `gamlss.family` object which can be used to fit a `WALD` distribution in the `gamlss()` function.

### Author(s)

Sofía Cuartas García, <scuartasg@unal.edu.co>

## References

Heathcote, A. (2004). Fitting Wald and ex-Wald distributions to response time data: An example using functions for the S-PLUS package. *Behavior Research Methods, Instruments, & Computers*, 36, 678-694.

## See Also

[dWALD](#).

## Examples

```
# Example 1
# Generating random values with
# known mu and sigma
require(gamlss)
mu <- 1.5
sigma <- 4.0

y <- rWALD(10000, mu, sigma)

mod1 <- gamlss(y~1, sigma.fo=~1, family="WALD",
               control=gamlss.control(n.cyc=5000, trace=TRUE))

exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))

# Example 2
# Generating random values under some model

# A function to simulate a data set with Y ~ WALD
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(0.75 - 0.69 * x1) # Approx 1.5
  sigma <- exp(0.5 - 0.64 * x2) # Approx 1.20
  y <- rWALD(n, mu, sigma)
  data.frame(y=y, x1=x1, x2=x2)
}

dat <- gendat(n=200)

mod2 <- gamlss(y~x1, sigma.fo=~x2, family=WALD, data=dat,
               control=gamlss.control(n.cyc=5000, trace=TRUE))

summary(mod2)
```

---

WG

*The Weibull Geometric family*

---

### Description

The Weibull Geometric distribution

### Usage

```
WG(mu.link = "log", sigma.link = "log", nu.link = "logit")
```

### Arguments

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.  
`sigma.link` defines the `sigma.link`, with "log" link as the default for the `sigma`.  
`nu.link` defines the `nu.link`, with "log" link as the default for the `nu` parameter.

### Details

The weibull geometric distribution with parameters `mu`, `sigma` and `nu` has density given by

$$f(x) = (\sigma\mu^\sigma(1-\nu)x^{\sigma-1}\exp(-(\mu x)^\sigma))(1-\nu\exp(-(\mu x)^\sigma))^{-2},$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $0 < \nu < 1$ .

### Value

Returns a `gamlss.family` object which can be used to fit a WG distribution in the `gamlss()` function.

### Author(s)

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

### References

Barreto-Souza, W., de Morais, A. L., & Cordeiro, G. M. (2011). The Weibull-geometric distribution. *Journal of Statistical Computation and Simulation*, 81(5), 645-657.

### See Also

[dWG](#)

**Examples**

```

# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rWG(n=100, mu = 0.9, sigma = 2, nu = 0.5)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='WG',
              control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))

# Example 2
# Generating random values under some model
n <- 200
x1 <- runif(n)
x2 <- runif(n)
mu <- exp(- 0.2 * x1)
sigma <- exp(1.2 - 1 * x2)
nu <- 0.5
x <- rWG(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, mu.fo=~x1, sigma.fo=~x2, nu.fo=~1, family=WG,
              control=gamlss.control(n.cyc=50000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
coef(mod, what='nu')

```

---

 WGEE

*The Weighted Generalized Exponential-Exponential family*


---

**Description**

The Weighted Generalized Exponential-Exponential family

**Usage**

```
WGEE(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

`mu.link` defines the `mu.link`, with "log" link as the default for the `mu` parameter.

sigma.link defines the sigma.link, with "log" link as the default for the sigma.  
 nu.link defines the nu.link, with "log" link as the default for the nu parameter.

### Details

The Weighted Generalized Exponential-Exponential distribution with parameters  $\mu$ ,  $\sigma$  and  $\nu$  has density given by

$$f(x) = \sigma\nu \exp(-\nu x)(1 - \exp(-\nu x))^{\sigma-1}(1 - \exp(-\mu\nu x))/1 - \sigma B(\mu + 1, \sigma),$$

for  $x > 0$ ,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu > 0$ .

### Value

Returns a `gamlss.family` object which can be used to fit a WGEE distribution in the `gamlss()` function.

### Author(s)

Johan David Marin Benjumea, <johand.marin@udea.edu.co>

### References

Mahdavi, A. (2015). Two weighted distributions generated by exponential distribution. *Journal of Mathematical Extension*, 9, 1-12.

### See Also

[dWGEE](#)

### Examples

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rWGEE(n=1000, mu = 5, sigma = 0.5, nu = 1)

# Fitting the model
require(gamlss)

mod <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family='WGEE',
             control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod, what='mu'))
exp(coef(mod, what='sigma'))
exp(coef(mod, what='nu'))

# Example 2
# Generating random values under some model
n <- 500
x1 <- runif(n, min=0.4, max=0.6)
```

```
x2 <- runif(n, min=0.4, max=0.6)
mu <- exp(2 - x1)
sigma <- exp(1 - 3*x2)
nu <- 1
x <- rWGEE(n=n, mu, sigma, nu)

mod <- gamlss(x~x1, sigma.fo=~x2, nu.fo=~1, family=WGEE,
              control=gamlss.control(n.cyc=50000, trace=FALSE))

coef(mod, what="mu")
coef(mod, what="sigma")
exp(coef(mod, what="nu"))
```

WP

*The Weibull Poisson family***Description**

The Weibull Poisson family

**Usage**

```
WP(mu.link = "log", sigma.link = "log", nu.link = "log")
```

**Arguments**

<code>mu.link</code>	defines the <code>mu.link</code> , with "log" link as the default for the <code>mu</code> parameter.
<code>sigma.link</code>	defines the <code>sigma.link</code> , with "log" link as the default for the <code>sigma</code> .
<code>nu.link</code>	defines the <code>nu.link</code> , with "log" link as the default for the <code>nu</code> parameter.

**Details**

The Weibull Poisson distribution with parameters `mu`, `sigma` and `nu` has density given by

$$f(x) = \frac{\mu\sigma\nu e^{-\nu}}{1-e^{-\nu}} x^{\mu-1} \exp(-\sigma x^{\mu} + \nu \exp(-\sigma x^{\mu})),$$

for  $x > 0$ .

**Value**

Returns a `gamlss.family` object which can be used to fit a WP distribution in the `gamlss()` function.

**Author(s)**

Amylkar Urrea Montoya, <amylkar.urrea@udea.edu.co>

**References**

Lu, Wanbo, and Daimin Shi. "A new compounding life distribution: the Weibull–Poisson distribution." *Journal of applied statistics* 39.1 (2012): 21-38.

**See Also**[dWP](#)**Examples**

```
# Example 1
# Generating some random values with
# known mu, sigma and nu
y <- rWP(n=3000, mu=1.5, sigma=0.5, nu=0.5)

# Fitting the model
require(gamlss)

mod1 <- gamlss(y~1, sigma.fo=~1, nu.fo=~1, family=WP,
               control=gamlss.control(n.cyc=5000, trace=FALSE))

# Extracting the fitted values for mu, sigma and nu
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))
exp(coef(mod1, what="nu"))

# Example 2
# Generating random values for a regression model

# A function to simulate a data set with  $Y \sim WP$ 
gendat <- function(n) {
  x1 <- runif(n)
  x2 <- runif(n)
  mu <- exp(-1.3 + 3.1 * x1)
  sigma <- exp(0.9 - 3.2 * x2)
  nu <- 0.5
  y <- rWP(n=n, mu, sigma, nu)
  data.frame(y=y, x1=x1, x2)
}

set.seed(1234)
dat <- gendat(n=100)

# Fitting the model
mod2 <- NULL
mod2 <- gamlss(y~x1, sigma.fo=~x2, nu.fo=~1,
               family=WP, data=dat,
               control=gamlss.control(n.cyc=5000, trace=FALSE))

coef(mod2, what="mu")
coef(mod2, what="sigma")
exp(coef(mod2, what="nu"))
```

# Index

- \* **datasets**
  - equipment, [142](#)
  - mice, [169](#)
- \* **initValOW**
  - initValuesOW, [161](#)
- AddW, [4, 33](#)
- all, [177](#)
- BGE, [6, 35](#)
- BS, [7, 37](#)
- BS10, [10, 40](#)
- BS11, [12, 42](#)
- BS12, [13, 45](#)
- BS13, [15, 47](#)
- BS2, [17, 49](#)
- BS3, [19, 52](#)
- BS4, [21, 54](#)
- BS5, [24, 56](#)
- BS6, [25, 58](#)
- BS7, [27, 61](#)
- CJ2, [29, 63](#)
- CS2e, [30, 66](#)
- dAddW, [5, 32](#)
- dBGE, [7, 34](#)
- dBBS, [8, 36](#)
- dBBS10, [11, 39](#)
- dBBS11, [13, 41](#)
- dBBS12, [14, 44](#)
- dBBS13, [16, 46](#)
- dBBS2, [18, 48](#)
- dBBS3, [20, 51](#)
- dBBS4, [22, 53](#)
- dBBS5, [24, 55](#)
- dBBS6, [26, 57](#)
- dBBS7, [28, 60](#)
- dCJ2, [29, 62](#)
- dCS2e, [31, 65](#)
- dEEG, [67, 136](#)
- dEGG, [69, 138](#)
- dEMWEx, [71, 139](#)
- dEOFNH, [73, 141](#)
- dEW, [75, 144](#)
- dEXL, [76](#)
- dExW, [80, 148](#)
- dExWALD, [81, 150](#)
- dFWE, [84](#)
- dGammaW, [86, 153](#)
- dGGD, [88, 154](#)
- dGIW, [90, 156](#)
- dGL2, [92, 158](#)
- dGMW, [94, 160](#)
- dGWF, [96](#)
- dIW, [98, 164](#)
- dKumIW, [100, 165](#)
- dLIN, [102](#)
- dLW, [104, 168](#)
- dMOEIW, [106, 170](#)
- dMOEW, [108, 172](#)
- dMOK, [109, 174](#)
- dMW, [111, 175](#)
- dNEE, [113, 179](#)
- dOW, [116, 189](#)
- dPL, [118, 185](#)
- dQXGP, [119, 186](#)
- dRNMW, [121](#)
- dRW, [124, 188](#)
- dSZMW, [126, 190](#)
- dWALD, [127, 192](#)
- dWG, [130, 193](#)
- dWGEE, [132, 195](#)
- dWP, [133, 197](#)
- EEG, [68, 135](#)
- EGG, [70, 137](#)
- EMWEx, [72, 139](#)
- EOFNH, [74, 140](#)
- equipment, [142](#)

- estim\_mu\_sigma\_GL2, 143  
 EW, 75, 143  
 EXL, 77, 144, 145  
 ExW, 81, 147  
 ExWALD, 83, 149  
  
 formula, 162  
 FWE, 85, 151  
  
 gamlss, 183  
 GammaW, 87, 152  
 GGD, 89, 154  
 GIW, 91, 155  
 GL2, 93, 157  
 GMW, 95, 160  
  
 hAddW (dAddW), 32  
 hBGE (dBGE), 34  
 hBS (dBS), 36  
 hBS10 (dBS10), 39  
 hBS11 (dBS11), 41  
 hBS12 (dBS12), 44  
 hBS13 (dBS13), 46  
 hBS2 (dBS2), 48  
 hBS3 (dBS3), 51  
 hBS4 (dBS4), 53  
 hBS5 (dBS5), 55  
 hBS6 (dBS6), 57  
 hBS7 (dBS7), 60  
 hCJ2 (dCJ2), 62  
 hCS2e (dCS2e), 65  
 hEEG (dEEG), 67  
 hEGG (dEGG), 69  
 hEMWEx (dEMWEx), 71  
 hEOFNH (dEOFNH), 73  
 hEW (dEW), 75  
 hEXL (dEXL), 76  
 hExW (dExW), 80  
 hFWE (dFWE), 84  
 hGammaW (dGammaW), 86  
 hGGD (dGGD), 88  
 hGIW (dGIW), 90  
 hGL2 (dGL2), 92  
 hGMW (dGMW), 94  
 hGWF (dGWF), 96  
 hIW (dIW), 98  
 hKumIW (dKumIW), 100  
 hLIN (dLIN), 102  
 hLW (dLW), 104  
  
 hMOEIW (dMOEIW), 106  
 hMOEW (dMOEW), 108  
 hMOK (dMOK), 109  
 hMW (dMW), 111  
 hNEE (dNEE), 113  
 hOW (dOW), 116  
 hPL (dPL), 118  
 hQXGP (dQXGP), 119  
 hRNMW (dRNMW), 121  
 hRW (dRW), 124  
 hSZMW (dSZMW), 126  
 hWG (dWG), 130  
 hWGEE (dWGEE), 132  
 hWP (dWP), 133  
  
 initValuesOW, 161, 177, 183, 189  
 interp.options, 162  
 IW, 99, 163  
  
 KumIW, 101, 164  
  
 LIN, 103, 166  
 loess.options, 162  
 logLik\_GL2, 167  
 LW, 105, 168  
  
 mice, 169  
 MOEIW, 107, 170  
 MOEW, 109, 171  
 MOK, 111, 173  
 MW, 113, 175  
 myOW\_region, 176  
  
 NEE, 114, 178  
  
 OW, 117, 177, 181, 189  
  
 pAddW (dAddW), 32  
 param.startOW, 183  
 pBGE (dBGE), 34  
 pBS (dBS), 36  
 pBS10 (dBS10), 39  
 pBS11 (dBS11), 41  
 pBS12 (dBS12), 44  
 pBS13 (dBS13), 46  
 pBS2 (dBS2), 48  
 pBS3 (dBS3), 51  
 pBS4 (dBS4), 53  
 pBS5 (dBS5), 55  
 pBS6 (dBS6), 57

- pBS7 (dBS7), [60](#)  
 pCJ2 (dCJ2), [62](#)  
 pCS2e (dCS2e), [65](#)  
 pEEG (dEEG), [67](#)  
 pEGG (dEGG), [69](#)  
 pEMWEx (dEMWEx), [71](#)  
 pEOFNH (dEOFNH), [73](#)  
 pEW (dEW), [75](#)  
 pEXL (dEXL), [76](#)  
 pExW (dExW), [80](#)  
 pExWALD (dExWALD), [81](#)  
 pFWE (dFWE), [84](#)  
 pGammaW (dGammaW), [86](#)  
 pGGD (dGGD), [88](#)  
 pGIW (dGIW), [90](#)  
 pGL2 (dGL2), [92](#)  
 pGMW (dGMW), [94](#)  
 pGWF (dGWF), [96](#)  
 pIW (dIW), [98](#)  
 pKumIW (dKumIW), [100](#)  
 PL, [119](#), [184](#)  
 pLIN (dLIN), [102](#)  
 pLW (dLW), [104](#)  
 pMOEIW (dMOEIW), [106](#)  
 pMOEW (dMOEW), [108](#)  
 pMOK (dMOK), [109](#)  
 pMW (dMW), [111](#)  
 pNEE (dNEE), [113](#)  
 pOW (dOW), [116](#)  
 pPL (dPL), [118](#)  
 pQXGP (dQXGP), [119](#)  
 pRNMW (dRNMW), [121](#)  
 pRW (dRW), [124](#)  
 pSZMW (dSZMW), [126](#)  
 pWALD (dWALD), [127](#)  
 pWG (dWG), [130](#)  
 pWGEE (dWGEE), [132](#)  
 pWP (dWP), [133](#)  
  
 qAddW (dAddW), [32](#)  
 qBGE (dBGE), [34](#)  
 qBS (dBS), [36](#)  
 qBS10 (dBS10), [39](#)  
 qBS11 (dBS11), [41](#)  
 qBS12 (dBS12), [44](#)  
 qBS13 (dBS13), [46](#)  
 qBS2 (dBS2), [48](#)  
 qBS3 (dBS3), [51](#)  
 qBS4 (dBS4), [53](#)  
  
 qBS5 (dBS5), [55](#)  
 qBS6 (dBS6), [57](#)  
 qBS7 (dBS7), [60](#)  
 qCJ2 (dCJ2), [62](#)  
 qCS2e (dCS2e), [65](#)  
 qEEG (dEEG), [67](#)  
 qEGG (dEGG), [69](#)  
 qEMWEx (dEMWEx), [71](#)  
 qEOFNH (dEOFNH), [73](#)  
 qEW (dEW), [75](#)  
 qEXL (dEXL), [76](#)  
 qExW (dExW), [80](#)  
 qExWALD (dExWALD), [81](#)  
 qFWE (dFWE), [84](#)  
 qGammaW (dGammaW), [86](#)  
 qGGD (dGGD), [88](#)  
 qGIW (dGIW), [90](#)  
 qGL2 (dGL2), [92](#)  
 qGMW (dGMW), [94](#)  
 qGWF (dGWF), [96](#)  
 qIW (dIW), [98](#)  
 qKumIW (dKumIW), [100](#)  
 qLIN (dLIN), [102](#)  
 qLW (dLW), [104](#)  
 qMOEIW (dMOEIW), [106](#)  
 qMOEW (dMOEW), [108](#)  
 qMOK (dMOK), [109](#)  
 qMW (dMW), [111](#)  
 qNEE (dNEE), [113](#)  
 qOW (dOW), [116](#)  
 qPL (dPL), [118](#)  
 qQXGP (dQXGP), [119](#)  
 qRNMW (dRNMW), [121](#)  
 qRW (dRW), [124](#)  
 qSZMW (dSZMW), [126](#)  
 qWALD (dWALD), [127](#)  
 qWG (dWG), [130](#)  
 qWGEE (dWGEE), [132](#)  
 qWP (dWP), [133](#)  
 QXGP, [121](#), [186](#)  
  
 rAddW (dAddW), [32](#)  
 rBGE (dBGE), [34](#)  
 rBS (dBS), [36](#)  
 rBS10 (dBS10), [39](#)  
 rBS11 (dBS11), [41](#)  
 rBS12 (dBS12), [44](#)  
 rBS13 (dBS13), [46](#)  
 rBS2 (dBS2), [48](#)

rBS3 (dBS3), 51  
rBS4 (dBS4), 53  
rBS5 (dBS5), 55  
rBS6 (dBS6), 57  
rBS7 (dBS7), 60  
rCJ2 (dCJ2), 62  
rCS2e (dCS2e), 65  
rEEG (dEEG), 67  
rEGG (dEGG), 69  
rEMWEx (dEMWEx), 71  
rEOFNH (dEOFNH), 73  
rEW (dEW), 75  
rEXL (dEXL), 76  
rExW (dExW), 80  
rExWALD (dExWALD), 81  
rFWE (dFWE), 84  
rGammaW (dGammaW), 86  
rGGD (dGGD), 88  
rGIW (dGIW), 90  
rGL2 (dGL2), 92  
rGMW (dGMW), 94  
rGWF (dGWF), 96  
rIW (dIW), 98  
rKumIW (dKumIW), 100  
rLIN (dLIN), 102  
rLW (dLW), 104  
rMOEIW (dMOEIW), 106  
rMOEW (dMOEW), 108  
rMOK (dMOK), 109  
rMW (dMW), 111  
rNEE (dNEE), 113  
rOW (dOW), 116  
rPL (dPL), 118  
rQXGP (dQXGP), 119  
rRNMW (dRNMW), 121  
rRW (dRW), 124  
rSZMW (dSZMW), 126  
RW, 125, 187  
rWALD (dWALD), 127  
rWG (dWG), 130  
rWGEE (dWGEE), 132  
rWP (dWP), 133

summary.initValOW, 189  
SZMW, 127, 189

TTTE\_Analytical, 162

WALD, 128, 191  
WG, 131, 193  
WGEE, 133, 194  
WP, 134, 196